

MATH 2331  
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Practice Exam I

1. Given matrices  $A \in \mathbb{R}^{3 \times 5}$ ,  $B \in \mathbb{R}^{5 \times 3}$ ,  $C \in \mathbb{R}^{5 \times 1}$ ,  $D \in \mathbb{R}^{3 \times 1}$ . Which of the following matrix operations are allowed?
- (a)  $BAD$ ;
  - (b)  $A(B + C)$ ;
  - (c)  $AC + BD$ ;
  - (d)  $ABD$ .

2. Find the determinant of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ ?

3. Find the inverse of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ ?

4. Find the inverse, assuming it exists, of the block matrix  $A = \begin{bmatrix} I & 0 \\ C & B \end{bmatrix}$ ?

5. Consider the system of equations:

$$\begin{cases} -3x + 4y = 8 \\ 6x + ty = s \end{cases}$$

where  $t$  and  $s$  are real numbers.

- (a) Find values for  $t$  and  $s$  so that the system has exactly one solution.
  - (b) Find values for  $t$  and  $s$  so that the system has no solutions.
  - (c) Find values for  $t$  and  $s$  so that the system has infinitely many solutions.
6. True or False:
- (a)  $U^{-T} = U$ ?
  - (b)  $(A + B)^T = B^T + A^T$ ?
  - (c)  $P^{-1} = P$  where  $P$  is permutation matrix?
  - (d)  $(AB)^{-1} = AB$  if  $A$  and  $B$  are symmetric?
  - (e) If  $3A^2 = I$  then  $A^{-1} = 3A$ ?
  - (f)  $A = \begin{bmatrix} -1 & 2 & 1 \\ -2 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$  is symmetric?
  - (g) If  $A$  is invertible then  $A^2$  and  $A^{-1}$  are invertible?
  - (h) If  $A$  is invertible then it is necessarily square.

7. Find the inverse of  $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$  if exists.

8. Find inverses of  $E_{13}(-1) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $E_2(3) = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$  if exist.

9. Find a solution to the following system:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 = 4 \\ 5x_2 + 2x_3 - x_4 = 3 \\ 3x_2 + 2x_4 = 10 \end{cases}$$

10. Calculate  $(ABC)^T$  for the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 \\ 3 & -1 \end{bmatrix}.$$

11. Transform the system:

$$\begin{cases} 2x + 4y - 2z = 2 \\ 4x + 9y - 3z = 8 \\ -2x - 3y + 7z = 10 \end{cases}$$

into triangular form and construct all elimination matrices for such a transformation.

12. Construct a  $2 \times 2$  matrix  $A$  transforming  $(x, y)$  into  $(2x, 3y)$ .

13. Solve the system:

$$\begin{cases} 2x + 3y = -2 \\ x - y = -6 \end{cases}$$