

1. Is the set  $V$  of positive real numbers (i.e.  $x > 0$ ) with addition and scalar multiplication defined as follows:

$$u + v = uv, \quad cu = u^c,$$

a vector space? Why?

2. Is the set  $V$  of two-dimensional vectors  $v$  with the standard scalar multiplication and vector addition and defined by

$$u + v = (u_1, u_2) + (v_1, v_2) = (u_1 + 2v_1, u_2 + v_2)$$

a vector space? Why?

3. Consider the set  $W = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 0\}$ . Is  $W$  a subspace of  $\mathbb{R}^3$ ?

4. Let  $M$  be the space of  $2 \times 2$  matrices. Consider the set  $W = \left\{ A \in M : A = \begin{bmatrix} 1 & a_{12} \\ 0 & a_{22} \end{bmatrix} \right\}$ . Is  $W$  a subspace of  $M$ ?

5. Find all special solutions to

$$\begin{cases} x_1 - x_2 + 2x_3 = 0 \\ 2x_1 + x_2 + x_3 = 0 \\ x_1 + x_2 = 0 \end{cases}$$

6. Construct a nullspace matrix  $N$  for the matrix  $A$  such that its reduced row echelon matrix is:

(a)  $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ ?

(b)  $R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ?

7. For matrices

$$A = \begin{bmatrix} 2 & 6 & 4 \\ -1 & 2 & 3 \\ 3 & -1 & -4 \\ 4 & 3 & 5 \\ 2 & 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 9 & 1 & -3 & 6 \\ -1 & -6 & 1 & 1 & -2 \\ 3 & -2 & 1 & 4 & 1 \\ 1 & 3 & 0 & 2 & 4 \\ 0 & 2 & 0 & 1 & 3 \\ 3 & 1 & 1 & 2 & 2 \end{bmatrix}$$

describe their column and null- spaces.

8. True or False (with reason if true and example if false):

(a)  $C(A) = \mathbb{R}^2$  for  $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ ?

(b)  $N(A) = \{\mathbf{0}\}$  for  $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ ?

(c) Vectors  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$  are linearly independent?

(d) The reduced row echelon matrix for  $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$  is  $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ ?

(e) The rank of  $A$  from (b) is  $r = 2$ ?

(f) If  $n > m$  then there are at least one "free" variable?

(g) The number of special solutions to  $A\mathbf{x} = \mathbf{0}$  with  $A \in \mathbb{R}^{m \times n}$  is in general equal to  $m$ ?

(h)  $C(A) = C(R)$  where  $R$  is the reduced row echelon matrix for  $A$ ?

(i)  $N(A) = C(R)$  where  $R$  is the reduced row echelon matrix for  $A$ ?

(j) The rank of  $A$  is the number of nonzero rows in  $R$ ?

(k)  $C(A) = C(2A)$ ?

9. Is the following set of vectors linearly dependent or independent:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 5 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 6 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} -6 \\ 7 \\ -1 \\ 1 \\ 1 \end{bmatrix} ?$$