MATH 2331
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Practice Exam II

1. Is the set V of positive real numbers (i.e. $x > 0$) with addition and scalar multiplication defined as follows:

$$u + v = uv, \quad cu = u^c,$$

a vector space? Why?
Answer: It is a vector space due to all 8 axioms!

2. Is the set V of two-dimensional vectors $\boldsymbol{v}$ with the standard scalar multiplication and vector addition and defined by

$$\boldsymbol{u} + \boldsymbol{v} = (u_1, u_2) + (v_1, v_2) = (u_1 + 2v_1, u_2 + v_2)$$

a vector space? Why?
Answer: It is not a space since the commutative law of addition does not hold

3. Consider the set $W = \{\boldsymbol{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 : \ x_1 = 0\}$. Is W a subspace of $\mathbb{R}^3$?
Answer: Yes, it is a subspace due to 3 properties of it

4. Let $\boldsymbol{M}$ be the space of 2×2 matrices. Consider the set $W = \left\{ A \in \boldsymbol{M} : \ A = \begin{bmatrix} 1 & a_{12} \\ 0 & a_{22} \end{bmatrix} \right\}$. Is W a subspace of $\boldsymbol{M}$?
Answer: No, it is not a subspace since the zero element is not in it

5. Find all special solutions to

$$\begin{cases} x_1 & -x_2 & +2x_3 & = 0 \\ 2x_1 & +x_2 & +x_3 & = 0 \\ x_1 & +x_2 & & = 0 \end{cases}$$

Answer: $[R|0] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ hence $\boldsymbol{s} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

6. Construct a nullspace matrix N for the matrix A such that its reduced row echelon matrix is:
(a) $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$?

Answer: since the number of special solutions is $n - r = 2 - 2 = 0$ hence N is empty
(b) $R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$?

Answer: $N = \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ since the number of columns in N is $n - r = 4 - 2 = 2$

7. For matrices

$$A = \begin{bmatrix} 2 & 6 & 4 \\ -1 & 2 & 3 \\ 3 & -1 & -4 \\ 4 & 3 & 5 \\ 2 & 4 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 9 & 1 & -3 & 6 \\ -1 & -6 & 1 & 1 & -2 \\ 3 & -2 & 1 & 4 & 1 \\ 1 & 3 & 0 & 2 & 4 \\ 0 & 2 & 0 & 1 & 3 \\ 3 & 1 & 1 & 2 & 2 \end{bmatrix}$$

describe their column and null- spaces.

Answer: $R_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ hence $N(A) = \{\mathbf{0}\} \subset \mathbb{R}^3$

$R_B = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ hence $N(B) = \{\boldsymbol{x} \in \mathbb{R}^5 : \boldsymbol{x} = x_6 \begin{bmatrix} 1 \\ -1 \\ -2 \\ -1 \\ 1 \end{bmatrix}\}$

8. True or False (with reason if true and example if false):

(a) $C(A) = \mathbb{R}^2$ for $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$?

Answer: F

(b) $N(A) = \{\mathbf{0}\}$ for $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$?

Answer: F

(c) Vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ are linearly independent?

Answer: F

(d) The reduced row echelon matrix for $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ is $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$?

Answer: T

(e) The rank of A from (b) is $r = 2$?

Answer: T

(f) If $n > m$ then there are at least one "free" variable?

Answer: T

(g) The number of special solutions to $A\boldsymbol{x} = \mathbf{0}$ with $A \in \mathbb{R}^{m \times n}$ is in general equal to m?

Answer: F

(h) $C(A) = C(R)$ where R is the reduced row echelon matrix for A?

Answer: F

(i) $N(A) = C(R)$ where R is the reduced row echelon matrix for A?

Answer: F

(j) The rank of A is the number of nonzero rows in R?

Answer: T

(k) $C(A) = C(2A)$?

Answer: T

9. Is the following set of vectors linearly dependent or independent:

$$\boldsymbol{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 5 \\ 2 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 6 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_4 = \begin{bmatrix} -6 \\ 7 \\ -1 \\ 1 \\ 1 \end{bmatrix}?$$

Answer: $A = \begin{bmatrix} 2 & 1 & 2 & -6 \\ -1 & 2 & 1 & 7 \\ 3 & -1 & -3 & -1 \\ 1 & 5 & 6 & 1 \\ 2 & 2 & 1 & 1 \end{bmatrix} \Rightarrow R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ hence linearly independent