MATH 2331
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Practice Final Exam

1. True or False (with reason if true and example if false):

   (a) $\det(A+B) = \det A + \det B$?

   (b) $\det A^{-T} = \dfrac{1}{\det A}$?

   (c) If λ_1, λ_2 are eigenvalues of A then $\lambda_1 + \lambda_2$ is also an eigenvalue of A?

   (d) Let R be the reduced row echelon matrix for A. Then in general their determinants coincide?

   (e) If $\lambda = \alpha + i\beta$ is an eigenvalue of A with the corresponding eigenvector $\boldsymbol{v}$ then $\overline{\lambda} = \alpha - i\beta$ is also an eigenvalue of A with the same eigenvector $\boldsymbol{v}$?

   (f) If $\boldsymbol{x}$ is an eigenvector of A corresponding to an eigenvalue λ then $c\boldsymbol{x}$, where c is any scalar, is also an eigenvector of A corresponding to the same eigenvalue λ?

   (g) $\det A^T = -\det A$?

   (h) $\det 2A = 2\det A$?

   (i) If λ is an eigenvalue of A then $1/\lambda$ is an eigenvalue of A^{-1}?

   (j) A matrix A is singular if and only if $\det A = 0$?

2. For $A \in \mathbb{R}^{2\times 2}$ satisfying

$$A\begin{bmatrix}1\\ i\end{bmatrix} = \begin{bmatrix}-i\\ 1\end{bmatrix}$$

   find all eigenvalues and corresponding eigenvectors.

3. For the matrix

$$A = \begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$$

   (a) compute $\det A$;

   (b) find all eigenvalues and corresponding eigenvectors.

4. Solve $A\boldsymbol{x} = \boldsymbol{b}$ using the **Cramer's Rule** for

$$\text{(a)}\quad A = \begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix}2\\ -1\end{bmatrix}$$

$$\text{(b)}\quad A = \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 1 & 1 & -2\end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix}2\\ -1\\ -4\end{bmatrix}$$

5. Find all eigenvalues and corresponding eigenvectors for

   (a) $\begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix}$

   (b) $\begin{bmatrix}2 & 3\\ 2 & 1\end{bmatrix}$

(c) $\begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

6. Given an eigenvector $\boldsymbol{x}$ of the matrix A. How to determine the associated eigenvalue λ?

7. Given the two eigenvalues of $A \in \mathbb{R}^{2\times 2}$: $\lambda_1 = 2$, $\lambda_2 = 4$. Construct the characteristic equation:

$$\det(A - \lambda I) = 0.$$

8. Find eigenvalues of A^{-1} if:

(a) $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$

9. Compute $\det A$ of the following matrix using the properties of determinants only:

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$