ADVANCED LINEAR ALGEBRA

REVIEW FOR TEST 2

Know the Definitions. Vector space. Subspace. Subspace spanned by a set of vectors. Solution space of a homogeneous system of linear equations. Row space of a matrix. Linearly dependent set of vectors. Linearly independent set of vectors. Basis. Finite dimensional vector space. Standard basis for F^n. Dimension of a finite–dimensional vector space. Ordered basis. Coordinates of a vector α relative to an ordered basis $\mathcal{B}$. Coordinate matrix $[\alpha]_{\mathcal{B}}$ of a vector α relative to an ordered basis $\mathcal{B}$. Row rank of a matrix.

Know Examples. Know examples for vector spaces: F^n (know the geometric interpretation of vector addition and scalar multiplication for $\mathbf{R^2}$ and $\mathbf{R^3}$), $F^{m\times n}$, the space of polynomial functions over F. Know examples of subspaces. Know examples of subsets of a vector space which are not subspaces. Know examples of linearly independent sets and of linearly dependent sets. Know examples of different bases for a vector space V (e.g. for $V = \mathbf{R^2}$, $V = \mathbf{R^3}$, $V = F^n$, and $V = F^{m\times n}$). Know examples of vector spaces that are not finite–dimensional (hint: the space of all polynomial functions over F).

Know the Results. A nonempty subset W of a vector space V over F is a subspace iff for any vectors $\alpha, \beta \in W$ and for each scalar $c \in F$, the vector $c\alpha + \beta$ belongs to W (Theorem 1, page 35). The subspace spanned by a nonempty set S of vectors in V is the set of all linear combinations of vectors in S (Theorem 2, page 37). If a vector space V has a spanning set consisting of m vectors in V, then every linearly independent set of vectors in V contains at most m vectors (Theorem 4, page 44). If V is a finite–dimensional vector space, then any two bases for V contain the same number of elements (Corollary 1, page 44). Given a linearly independent subset S of a vector space V, if $\alpha \in V$ such that α is not in the span of S, then the set $S \cup \{\alpha\}$ is linearly independent (Lemma, page 45). In a finite–dimensional vector space V, every linearly independent subset of V can be enlarged to a basis (Corollary 2, page 46). If the row vectors of an $n \times n$ matrix A over F are linearly independend, then A is invertible (Corollary 3, page 46). Given an n–dimensional vector space V and two ordered bases $\mathcal{B}$ and $\mathcal{B}'$ of V containing the vectors $\alpha_1, \ldots, \alpha_n$ and $\alpha'_1, \ldots, \alpha'_n$ (in this order!), respectively, there exists a unique $n \times n$ matrix P such that, for each $\beta \in V$, $[\beta]_{\mathcal{B}} = P[\beta]_{\mathcal{B}'}$; moreover, P is invertible and the jth column of P contains the coordinate matrix of α'_j with respect to the ordered basis $\mathcal{B}$ (Theorem 7, page 52). Given any n–dimensional vector space V over F with ordered basis $\mathcal{B}$, and given any invertible $n \times n$ matix P over F, there exists a unique ordered basis $\mathcal{B}'$ for V such that $[\beta]_{\mathcal{B}} = P[\beta]_{\mathcal{B}'}$ for each $\beta \in V$ (Theorem 8, page 53). Row–equivalent matrices have the same row space (Theorem 9, page 56). The nonzero rows of a row–reduced echelon matrix R form a basis for the row space of R (Theorem 10, page 56). Each $m \times n$ matrix A over F

is row–equivalent to exactly one row–reduced echelon matrix (1^{st} Corollary, page 58). Two $m \times n$ matrices over F are row–equivalent iff they have the same row space (2^{nd} Corollary, page 58).

Know how to work the homework problems. If you have questions, see me or see Maureen Royce.

Test 2 is scheduled for Wednesday, October 29, and will cover Chapter 2.