1.3 Fractions

GCF (Greatest Common Factor)

1. Write each of the given numbers as a product of prime factors.
2. The GCF of two or more numbers is the product of all prime factors **common** to every number.

Example: $10 = 2\cdot5$ and $8 = 2^3$.

$10 = 2\cdot5 \qquad 8 = 2^3$

GCF of 10 and 8 is: 2

Examples:

1. Find the GCF of 24 and 32.

$24 = 2^3\cdot3 = 2\cdot2\cdot2\cdot3$

$32 = 2^5 = 2\cdot2\cdot2\cdot2\cdot2$

$GCF(24,32) = \boxed{2^3}$

2. Find the GCF of 15 and 27.

$15 = 3\cdot5$

$27 = 3^3$

$GCF(15,27) = 3$

3. Find the GCF of 27, 18, and 45.

$27 = 3^3$

$18 = 2\cdot3^2$

$45 = 3^2\cdot5$

$GCF = \boxed{3^2 = 9}$

LCM (Least Common Multiple)

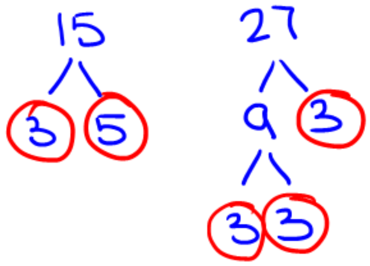
1. Write each of the given numbers as a product of prime factors.
2. Take the greatest power on each prime and multiply them.

Example: $10 = 2 \cdot 5$ and $8 = 2^3$.

LCM of 10 and 8 is: $2^3 \cdot 5 = 40$.

Examples:

1. Find the LCM of 15 and 27:

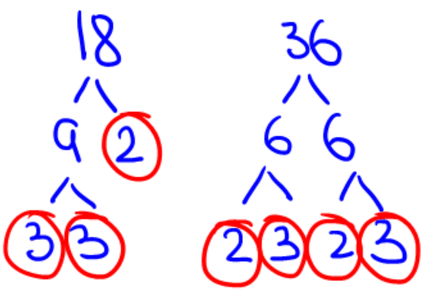


$15 = 3 \cdot 5$

$27 = 3^3$

$LCM = 3^3 \cdot 5 = 27 \cdot 5 = \boxed{135}$

2. Find the LCM of 18 and 36.

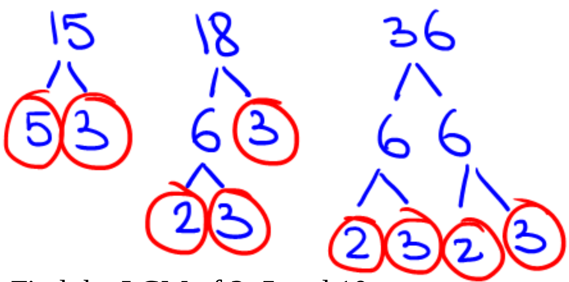


$18 = 2 \cdot 3^2$

$36 = 2^2 \cdot 3^2$

$LCM = 2^2 \cdot 3^2 = 4(9) = 36$

3. Find the LCM of 15, 18, and 36.



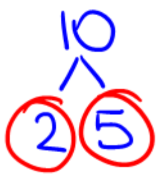
$15 = 3 \cdot 5$

$18 = 2 \cdot 3^2$

$36 = 2^2 \cdot 3^2$

$LCM = \boxed{2^2 \cdot 3^2 \cdot 5} = 180$

4. Find the LCM of 2, 5 and 10.



$2 = 2$

$5 = 5$

$10 = 2 \cdot 5$

$LCM = 2 \cdot 5 = 10$

Adding and Subtracting Fractions:

- Find a least common denominator using method for LCM
- Change the numerators of each fraction
- Add or subtract the numerators (keep denominator unchanged)
- Reduce

Examples:

1. $\frac{1}{4}+\frac{1}{5}=$ $\frac{1\cdot 5}{4\cdot 5}+\frac{1\cdot 4}{5\cdot 4}=\frac{5+4}{20}=\boxed{\frac{9}{20}}$

2. $\frac{5}{6}+\frac{3}{8}=$ $\frac{5\cdot 4}{6\cdot 4}+\frac{3\cdot 3}{8\cdot 3}=\frac{20+9}{24}=\frac{29}{24}=\boxed{1\frac{5}{24}}$

$24\overline{)29}$ → 1, -24, 5

$6, 8$

$6 = 2\cdot 3$

$8 = 2^3$

$LCM = 2^3\cdot 3 = 8\cdot 3 = 24$

3. $\frac{2\cdot 6}{5\cdot 6}+\frac{1\cdot 5}{6\cdot 5}+\frac{3\cdot 3}{10\cdot 3}=$

$=\frac{12+5+9}{30}=\frac{26}{30}=\boxed{\frac{13}{15}}$

4. $\frac{2}{5}-\frac{1}{6}=$

$\frac{2\cdot 6}{5\cdot 6}-\frac{1\cdot 5}{6\cdot 5}=\frac{12-5}{30}=\boxed{\frac{7}{30}}$

5. $3\frac{1}{5}-2\frac{1}{4}=$ $\frac{16\cdot 4}{5\cdot 4}-\frac{9\cdot 5}{4\cdot 5}=\frac{64-45}{20}=\boxed{\frac{19}{20}}$

$3\frac{1}{5}=\frac{3\cdot 5+1}{5}=\frac{16}{5}$ $\quad 2\frac{1}{4}=\frac{2\cdot 4+1}{4}=\frac{9}{4}$

6. $\frac{1}{2}+\frac{4}{5}-\frac{3}{10}=$ $\frac{1\cdot 5}{2\cdot 5}+\frac{4\cdot 2}{5\cdot 2}-\frac{3}{10}$

$=\frac{5+8-3}{10}=\frac{10}{10}=\boxed{1}$

$2=2$

$5=5$

$10=2\cdot 5$

$LCM=2\cdot 5=10$

7. $2\frac{1}{4}+3\frac{1}{5}-\frac{3}{10}= \frac{9\cdot 5}{4\cdot 5}+\frac{16\cdot 4}{5\cdot 4}-\frac{3\cdot 2}{10\cdot 2}=\frac{45+64-6}{20}=\frac{103}{20}=5\frac{3}{20}$

$2\frac{1}{4}=\frac{2\cdot 4+1}{4}=\frac{9}{4}$

$3\frac{1}{5}=\frac{3\cdot 5+1}{5}=\frac{16}{5}$

$$\begin{array}{r} 5 \\ 20\overline{)103} \\ -100 \\ \hline 3 \end{array}$$

8. $\frac{4}{5}+4= 4\frac{4}{5}$

$\frac{4}{5}+\frac{4\cdot 5}{1\cdot 5}=\frac{4+20}{5}=\frac{20}{5}=4\frac{4}{5}$

Multiplying and Dividing Fractions:

- Simplify the fractions if not in lowest terms.
- Multiply the numerators of the fractions to get the new numerator.
- Multiply the denominators of the fractions to get the new denominator.

Examples:

1. $\frac{1}{5}\times\frac{2}{3} = \frac{1\cdot 2}{5\cdot 3}=\boxed{\frac{2}{15}}$

2. $\frac{5}{\cancel{8}_{4}}\times\frac{\cancel{2}^{1}}{3}= \frac{5\cdot 1}{4\cdot 3}=\boxed{\frac{5}{12}}$

3. $\frac{4}{5}\times\frac{6}{1}= \frac{4\cdot 6}{5}=\frac{24}{5}=4\frac{4}{5}$

$$\begin{array}{r} 4 \\ 5\overline{)24} \\ -20 \\ \hline 4 \end{array}$$

$\frac{4}{5}\times 6 = 4\frac{4}{5}$

$\frac{4}{5}+6=6\frac{4}{5}$

Dividing Fractions:

- Multiply the first fraction by the reciprocal of the second

Examples:

1. $\frac{3}{2}\div\frac{6}{7}= \frac{\cancel{3}^{1}}{2}\cdot\frac{7}{\cancel{6}_{2}}=\frac{7}{4}=1\frac{3}{4}$

2. $\frac{4}{5} \div \frac{8}{11} =$ $\frac{\overset{1}{\cancel{4}}}{5} \cdot \frac{11}{\underset{2}{\cancel{8}}} = \frac{11}{10} = \boxed{1\frac{1}{10}}$

3. $\frac{4}{9} \div \frac{8}{1} =$ $\frac{\overset{1}{\cancel{4}}}{9} \cdot \frac{1}{\underset{2}{\cancel{8}}} = \boxed{\frac{1}{18}}$

$\frac{1}{2} = 1 \div 2$

4. $\frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{7}\right)} =$ $\frac{4}{5} \div \frac{2}{7} = \frac{\overset{2}{\cancel{4}}}{5} \cdot \frac{7}{\underset{1}{\cancel{2}}} = \frac{14}{5} = \boxed{2\frac{4}{5}}$

5. $\frac{\left(-\frac{7}{10}\right)}{\left(-\frac{2}{9}\right)} =$ $\frac{7}{10} \div \frac{2}{9} = \frac{7}{10} \cdot \frac{9}{2} = \frac{63}{20} = \boxed{3\frac{3}{20}}$