**Section 1.4: Exponents and Radicals**

Let n be a natural number. Then the exponential expression $x^n$ is defined by $x^n = \underbrace{x * x * x * \ldots * x}_{n \text{ times}}$.

$x^n$ is read as "$x$ to the $n$th power".

**Examples:** $2^4 = 2*2*2*2 = 16$, $(-3)^2 = (-3)(-3) = 9$

$4^3 = 4 \cdot 4 \cdot 4 = 64$ $(-5)^2 = (-5)(-5) = 25$ $-5^2 = -(5)(5) = -25$

**Rules for Exponents:**

Multiplying Powers: $a^m \times a^n = a^{m+n}$ $2^3 \times 2^2 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 2}_{3+2} = 2^5$

Dividing Powers: $\dfrac{a^m}{a^n} = a^{m-n}$ $\dfrac{2^5}{2^3} = 2^{5-3} = 2^2$

Negative Powers: $a^{-m} = \dfrac{1}{a^m}$ and $\dfrac{1}{a^{-n}} = a^n$ $2^{-3} = \dfrac{1}{2^3}$ $\dfrac{1}{2^{-5}} = 2^5$

Power Rule: $(a^m)^n = a^{mn}$ $(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$

Zero Power Rule: $a^0 = 1$ $(2014)^0 = 1$

*Note:* If no power is shown, then the exponent is 1. $2^1 = 2$

**Examples:** Simply having no negative exponents.

1. $(4^1)(4^3) = 4^{1+3} = \boxed{4^4}$

2. $(c^3 d^4)(c^5 d^2)$
$= c^{3+5} d^{4+2}$
$= \boxed{c^8 d^6}$

3. $\dfrac{a^5 b^{16} c^7}{a^9 b^8 c^{12}}$
$= a^{5-9} b^{16-8} c^{7-12}$
$= a^{-4} b^8 c^{-5}$
$= \boxed{\dfrac{b^8}{a^4 c^5}}$

4. $6x^{-3} = \boxed{\dfrac{6}{x^3}}$

$(6x)^{-3} = \dfrac{1}{(6x)^3} = \dfrac{1}{6^3 x^3}$

$$x^{-n} = \frac{1}{x^n} \qquad \frac{1}{x^{-n}} = x^n$$



5. $\frac{30e^{-4}f^3}{5(f^4)^{-1}} = \frac{\cancel{30}^{6}e^{-4}f^3}{\cancel{5}\cdot f^{-4}}$

$= \frac{6\,f^3\cdot f^4}{e^4} = \frac{6\,f^{3+4}}{e^4} = \boxed{\frac{6f^7}{e^4}}$

6. $\frac{3}{5^{-2}} = \boxed{3\cdot 5^2} = 3\cdot(25) = 75$

7. $\frac{y^{-6}}{y^{-8}}$

$= \frac{y^8}{y^6} = y^{8-6} = \boxed{y^2}$

8. $\left(\frac{3}{7}\right)^{-1} = \left(\frac{7}{3}\right)^1 = \boxed{\frac{7}{3}}$

9. $\frac{\cancel{12}^{2}x^3y^0z^{-4}}{\cancel{18}_{3}x^3y^{-3}z^4} = \frac{2y^3}{3z^4z^4}$

$= \boxed{\frac{2y^3}{3z^8}}$

10. $\left(\frac{4x^4}{16x^3y}\right)^{-1} = \frac{\cancel{16}^{4}x^3y}{\cancel{4}x^4}$

$= 4x^{3-4}y = 4x^{-1}y$

$= \boxed{\frac{4y}{x}}$

11. $\left(\frac{24x^3y^{-8}z^4}{476x^{-3}y^2z^4}\right)^0 = 1$

12. $(3^24^3)^8$

$= (3^2)^8(4^3)^8 = \boxed{3^{16}4^{24}}$

13. $(6a^2b^{-2}c^4)^2$

$= (6)^2(a^2)^2(b^{-2})^2(c^4)^2$

$= 36a^4b^{-4}c^8$

$= \boxed{\frac{36a^4c^8}{b^4}}$

14. $\frac{(mn^3)^{-2}}{(n^4)^2} = \frac{1}{(mn^3)^2(n^4)^2}$

$= \frac{1}{m^2n^6\cdot n^8}$

$= \boxed{\frac{1}{m^2n^{14}}}$

$(3x)^2 =$

A. $9x^2$
B. $6x^2$
C. $3x^2$

$(3x)^2 = (3)^2(x)^2 = 9x^2$

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**Simplifying Radicals**

$(-4)^2 = (-4)(-4) = 16$

$(4)^2 = (4)(4) = 16$

A number $y$ is called the **square root** of a number $x$ if $y^2 = x$.

$(-4)^2 = 4^2 = 16$. So, 4 and -4 are both square roots of 16.

In general, if $x > 0$, then $x$ has two square roots. However, we use the symbol $\sqrt{x}$ for the "principal square root", which is the positive square root of $x$.

**Examples:** Simplify the following.

4
9
16
25
36
49
64
81
100
121

1. $\sqrt{36} = 6$

2. $\sqrt{121} = 11$

3. $\sqrt{18} = \sqrt{9 \cdot 2}$

$= \sqrt{9} \cdot \sqrt{2}$

$= \boxed{3\sqrt{2}}$

4. $\sqrt{75}$

$= \sqrt{25 \cdot 3}$

$= \sqrt{25} \cdot \sqrt{3}$

$= \boxed{5\sqrt{3}}$

5. $\sqrt{10^2} = \boxed{10}$

$\sqrt{4^2} = 4$

$\sqrt{(2014)^2} = 2014$

6. $\sqrt{64} - 2^2$

$= 8 - 4$

$= \boxed{4}$

**Notation:** $x^{1/2} = \sqrt{x}$

7. $81^{1/2}$

$= \sqrt{81}$

$= 9$

8. $144^{1/2} + 49^{1/2} - \sqrt{169}$

$= \sqrt{144} + \sqrt{49} - \sqrt{169}$

$= 12 + 7 - 13$

$= \boxed{6}$