Math 1300 Section 1.7 Inequalities

Section 1.7: Solving Linear ~~Equations~~

An inequality is similar to an equation except instead of an equal sign "=" you find one of the following signs $<, >, \leq,$ and $\geq$.

- $<$ less than (the quantity to the left is less than the quantity to the right) $3<5$
- $\leq$ less than or equal to (the quantity to the left is less than or equal to the quantity to the right) $4 \leq 4$
- $>$ greater than (the quantity to the left is greater than the quantity to the right) $8>4$
- $\geq$ greater than or equal to (the quantity to the left is greater than or equal to the quantity to the right) $8 \geq 4$

To solve an inequality containing a variable, find all values of the variable that make the inequality true. In solving linear inequalities, isolate the variable on one side of the inequality symbol. A linear inequality has a solution that is over an interval and the answers are in what is called *interval notation*. This ensures that you have the complete answer.

Interval Notation: In interval notation a parentheses means does not include, while a bracket means it will be included.

Inequality	**Interval Notation**
$x < A$	$(-\infty, A)$
$x \leq A$	$(-\infty, A]$
$x > B$	(B, ∞)
$x \geq B$	$[B, \infty)$
$A < x < B$	(A, B)
$A \leq x \leq B$	$[A, B]$

any x $(-\infty, \infty)$

$x<5$ $(-\infty, 5)$ $x \leq 5$ $(-\infty, 5]$ $x>5$ $(5, \infty)$

Procedure: Get x on one side by itself (or in the case of compound inequalities, get x by itself in the middle). Operations you can use to leave x alone:

1. Add or subtract the SAME real number on either side (all sides)
2. Multiply both sides by the same POSITIVE real number
3. Divide both sides by the same POSITIVE real number.
4. Multiply or divide by a negative number AND SWITCH THE SIGN.

$x \geq 5$ $[5, \infty)$ $2<x<3$ $(2,3)$ $2 \leq x \leq 3$ $[2,3]$

Note: If you multiply or divide an inequality by a negative number you have to reverse the inequality sign for the solution to be correct.

$(-1)x > 5(-1)$ $-x < -5$

Examples:

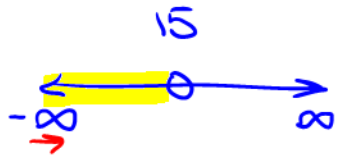
1. Express the solution for the ~~equation~~ inequality in interval notation.

$$x - 3 < 12$$

$+3 \quad +3$

$x < 15$

$(-\infty, 15)$



2. Express the solution for the ~~equation~~ inequality in interval notation.

$$3(x + 2) - 5 > 2(x - 4) + 16$$

$3x + 6 - 5 > 2x - 8 + 16$

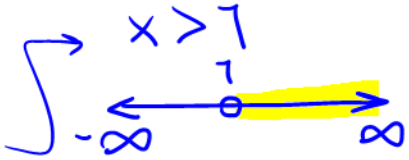
$3x + 1 > 2x + 8$

$-2x \quad -2x$

$x + 1 > 8$

$-1 \quad -1$

$x > 7$



$(7, \infty)$

3. Express the solution for the ~~equation~~ inequality in interval notation.

$$-\frac{4}{5}x + 6 \geq 10$$

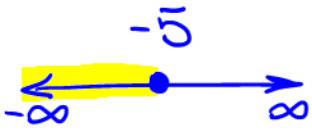
$-6 \quad -6$

$\left(-\frac{5}{4}\right)\left(-\frac{4}{5}\right)x \leq 4\left(-\frac{5}{4}\right)$ switch

$-4x \geq 20$

$\frac{-4x}{-4} \quad \frac{20}{-4}$

$x \leq -5$



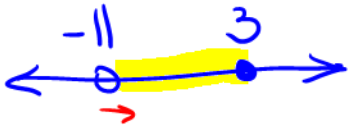
$(-\infty, -5]$

4. Express the solution for the equation in interval notation

$$-8 < 3 + x \leq 6$$

$-3 \quad -3 \quad -3$

$-11 < x \leq 3$



$(-11, 3]$

5. Express the solution for the equation in interval notation

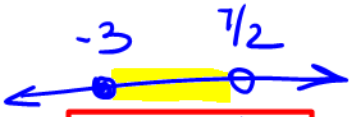
$$-4 < 3 - 2x \leq 9$$

$-3 \quad -3 \quad -3$

switch $-7 < -2x \leq 6$ switch

$\frac{-7}{-2} < \frac{-2x}{-2} \leq \frac{6}{-2}$

$7/2 > x \geq -3$



$[-3, 7/2)$

$[-3, 3\frac{1}{2})$

6. Express the solution for the equation in interval notation

LCD = 24

$$-\frac{7}{6} < \frac{-3(-x-1)}{8} < \frac{7}{3}$$

$4 \cdot 24 \quad 24 \quad 3 \quad 24 \quad 8$

$-28 < -9(-x-1) < 56$

$-28 < 9x + 9 < 56$

$-9 \quad -9 \quad -9$

$-37 < 9x < 47$

$\frac{-37}{9} < \frac{9x}{9} < \frac{47}{9}$

$-37/9 < x < 47/9$

$-4\frac{1}{9} < x < 5\frac{2}{9}$

$(-4\frac{1}{9}, 5\frac{2}{9})$

7. Express the solution for the equation in interval notation

$$(2)\,35 < 5x - \frac{5(x-7)}{2} \leq 70 \cdot 2$$

$70 < 10x - 5(x-7) \leq 140$

$70 < 10x - 5x + 35 \leq 140$

$70 < 5x + 35 \leq 140$

$-35 \quad -35 \quad -35$

$\frac{35}{5} < \frac{5x}{5} \leq \frac{105}{5}$

$7 < x \leq 21$

$(7, 21]$