Section 1.8: Solving Absolute Value Equations

|5| = 5

|-5| = 5

|0| = 0

Remember

The **absolute value** of *x*, denoted |*x*|, is the distance *x* is from 0.

Absolute Value Equations

To solve the equation $|x| = C$, use the following properties:

If C is positive, then |*x*| = C is equal to $x = \pm C$.

If C is negative, then |*x*| = C has no solution.

If C is zero, then the solution of |*x*| = 0 is $x = 0$.

|x| = 3 x = ±3 → x = 3, x = -3

|x| = -½ no solution

|x| = 0 x = 0

If the absolute value equation is more complicated than $|x| = C$, **isolate** the absolute value first and then solve it.

Examples:

1. Solve for x.

$|x| = 6$ 6 > 0

$x = \pm 6$

2. Solve for x.

$|2x - 3| = 7$ 7 > 0

$2x - 3 = \pm 7$

$2x - 3 = 7$ → $2x = 10$ → $x = 5$

$2x - 3 = -7$ → $2x = -4$ → $x = -2$

3. Solve for x.

$|6 - 2x| + 6 = 14$

$|6 - 2x| = 8$ 8 > 0

$6 - 2x = \pm 8$

$6 - 2x = 8$ → $-2x = 2$ → $x = -1$

$6 - 2x = -8$ → $-2x = -14$ → $x = 7$

4. Solve for x.

$2|-3(2x-8)|+4=30$

$-4 \quad -4$

$\frac{2|-3(2x-8)|}{2}=\frac{26}{2}$

$|-3(2x-8)|=13$

$-3(2x-8)=13$

$-6x+24=13$

$-24 \quad -24$

$\frac{-6x}{-6}=\frac{-11}{-6}$

$x=11/6$

$-3(2x-8)=-13$

$-6x+24=-13$

$-24 \quad -24$

$\frac{-6x}{-6}=\frac{-37}{-6}$

$x=37/6$

Has 2 solutions: $x=11/6$ and $x=37/6$

5. Solve for x.

$-4\left|\frac{1}{2}x+1\right|+3=-11$

$-3 \quad -3$

$\frac{-4\left|\frac{1}{2}x+1\right|}{-4}=\frac{-14}{-4}$

$\left|\frac{1}{2}x+1\right|=\frac{7}{2}$

$7/2>0$

$\frac{1}{2}x+1=\frac{7}{2}$

$-1 \quad -1$

$\frac{1}{2}x=\frac{7}{2}-\frac{2}{2}$

$(2)\frac{1}{2}x=\frac{5}{2}(2)$

$x=5$

$\frac{1}{2}x+1=-\frac{7}{2}$

$-1 \quad -1$

$\frac{1}{2}x=-\frac{7}{2}-\frac{1\cdot 2}{1\cdot 2}$

$\frac{1}{2}x=-\frac{7}{2}-\frac{2}{2}$

$(2)\frac{1}{2}x=-\frac{9}{2}(2)$

$x=-9$

2 Solutions:

$x=-9$

$x=5$

6. Solve for x.

$|2(x+3)-4|+5=4$

$|2(x+3)-4|+5=4$

$-5\ \ -5$

$|2(x+3)-4|=-1$

No solution

$|2(x+3)-4|+4=4$

$-4\ \ -4$

$|2(x+3)-4|=0$

$|x|=0$ $x=0$

$2(x+3)-4=0$

$+4\ \ +4$

$\frac{2(x+3)}{2}=\frac{4}{2}$

$x+3=2$

$-3\ \ -3$

$\boxed{x=-1}$