Section 2.2: The Distance and Midpoint Formula

For any two points **A** (x_1, y_1) and **B** (x_2, y_2), the distance between them is given by

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1: Find the distance between the following pair of points.

$x_1\ y_1 \quad x_2\ y_2$

a) $(-3,1)$ & $(1,3)$

$$d = \sqrt{(1-(-3))^2 + (3-1)^2} = \sqrt{(1+3)^2 + (3-1)^2}$$

$$= \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5}$$

$$= \boxed{2\sqrt{5}}$$

$x_1\ y_1 \quad x_2\ y_2$

b) $(-2, 5)$ & $(\frac{1}{2}, -1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\frac{1}{2} + \frac{2 \cdot 2}{1 \cdot 2} = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$$

$$d = \sqrt{\left(\frac{1}{2} - (-2)\right)^2 + (-1-5)^2} = \sqrt{\left(\frac{1}{2}+2\right)^2 + (-6)^2}$$

$$= \sqrt{\left(\frac{5}{2}\right)^2 + (-6)^2} = \sqrt{\frac{25}{4} + \frac{36 \cdot 4}{1 \cdot 4}} = \sqrt{\frac{25}{4} + \frac{144}{4}} = \sqrt{\frac{169}{4}} = \frac{\sqrt{169}}{\sqrt{4}} = \boxed{\frac{13}{2}}$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$= 6.5$$

$x_1\ y_1 \quad x_2\ y_2$

c) $(4, -6)$ & $\left(\frac{3}{2}, -2\right)$

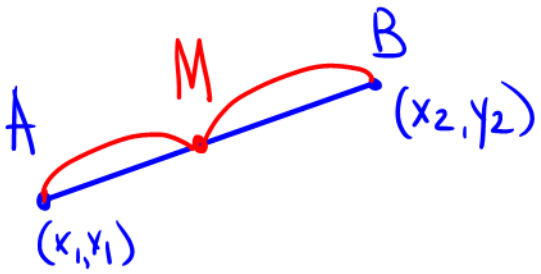
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{\left(\frac{3}{2} - 4\right)^2 + (-2-(-6))^2} = \sqrt{\left(\frac{-5}{2}\right)^2 + (-2+6)^2}$$

$$= \sqrt{\frac{25}{4} + (4)^2} = \sqrt{\frac{25}{4} + \frac{16 \cdot 4}{1 \cdot 4}} = \sqrt{\frac{25}{4} + \frac{64}{4}}$$

$$= \sqrt{\frac{89}{4}} = \frac{\sqrt{89}}{\sqrt{4}} = \boxed{\frac{\sqrt{89}}{2}}$$

$$\frac{3}{2} - \frac{4 \cdot 2}{1 \cdot 2} = \frac{3}{2} - \frac{8}{2} = \frac{-5}{2}$$



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Midpoint Formula

The midpoint of the line segment joining the two points **A** (x_1, y_1) and **B** (x_2, y_2) is given by

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Example 2: Find the midpoint between the following pair of points.

a) $(-3,1)$ & $(1,3)$

$x_1\ y_1 \quad x_2\ y_2$

$$M = \left(\frac{-3+1}{2}, \frac{1+3}{2}\right) = \boxed{(-1, 2)}$$

b) $(-2, -3)$ & $(4,6)$

$x_1\ y_1 \quad x_2\ y_2$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{-2+4}{2}, \frac{-3+6}{2}\right) = \boxed{\left(1, \frac{3}{2}\right) = (1, 1.5)}$$

c) $\left(-\frac{1}{2}, 2\right)$ & $\left(\frac{5}{2}, -6\right)$

$x_1\ y_1 \quad x_2\ y_2$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$-\frac{1}{2} + \frac{5}{2} = \frac{-1+5}{2} = \frac{4}{2} = 2$$

$$M = \left(\frac{-\frac{1}{2}+\frac{5}{2}}{2}, \frac{2+(-6)}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = \boxed{(1, -2)}$$

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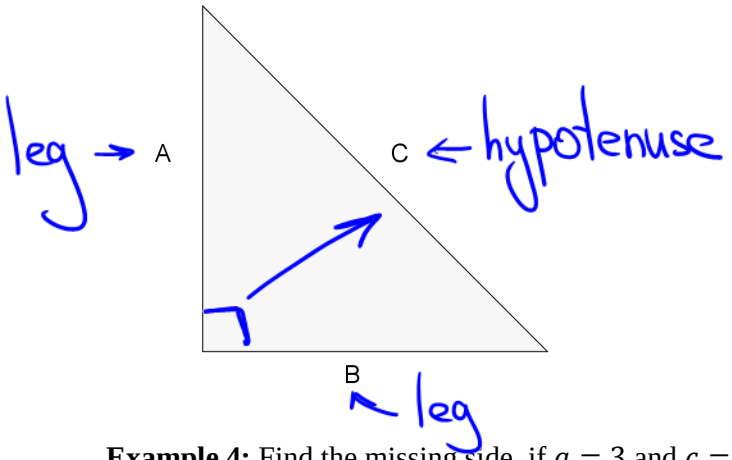
The **Pythagorean Theorem** states that in a right triangle, if a and b are the lengths of the legs, and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$

Note: To use the Pythagorean Theorem, you **must** have a right triangle

Example 3: Find the missing side, if $a = 6$ and $b = 8$.



$c = ?$

$a^2 + b^2 = c^2$

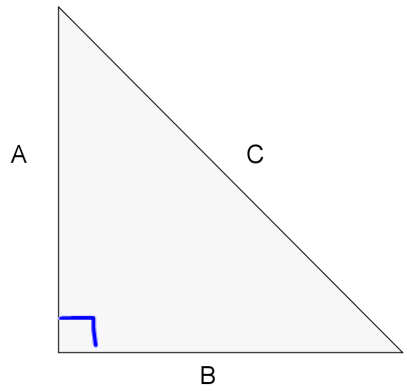
$(6)^2 + (8)^2 = c^2$

$36 + 64 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$ $\quad c = 10$

Example 4: Find the missing side, if $a = 3$ and $c = 5$.



$a^2 + b^2 = c^2$

$(3)^2 + b^2 = (5)^2$

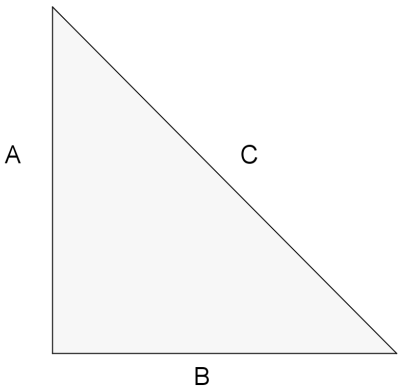
$9 + b^2 = 25$

$-9 \quad -9$

$\sqrt{b^2} = \sqrt{16}$

$b = 4$

Example 5: Find the missing side, if $a = 5$ and $c = 13$.



$a^2 + b^2 = c^2$

$(5)^2 + b^2 = (13)^2$

$25 + b^2 = 169$

$-25 \quad -25$

$\sqrt{b^2} = \sqrt{144}$

$b = 12$