

Section 2.5: Parallel and Perpendicular Lines

Two lines with slopes m_1 and m_2 are **parallel** if and only if $m_1 = m_2$

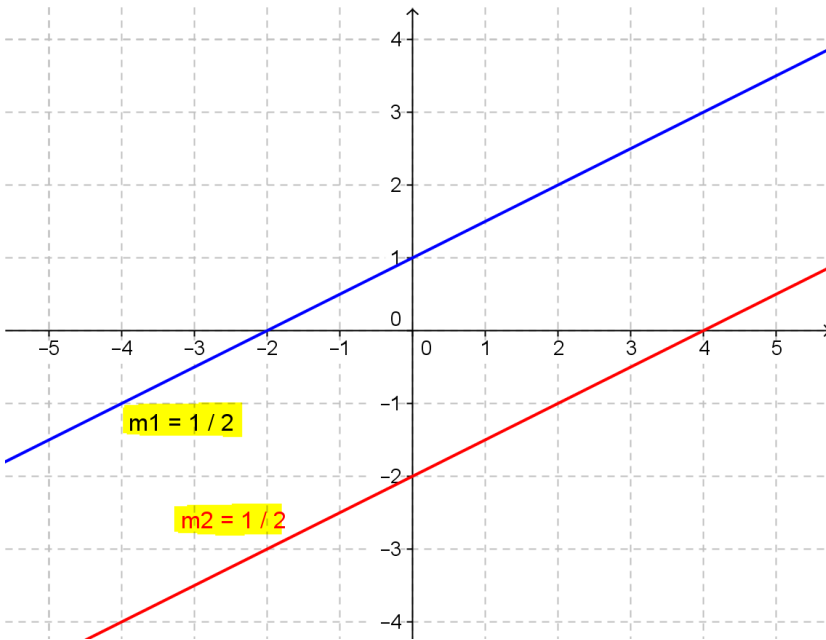
$m_1 = m_2$ AND

$b_1 = b_2$

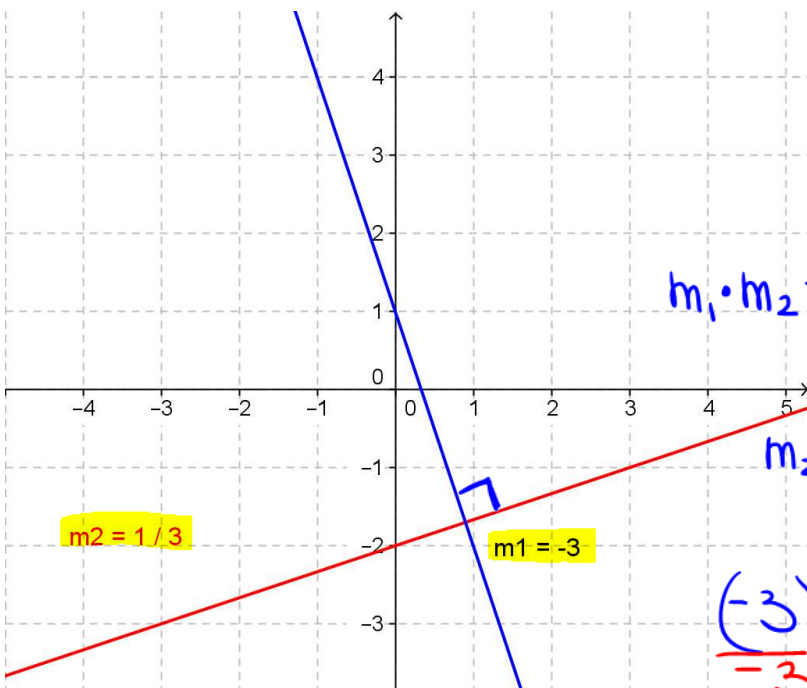
SAME

$y = \frac{1}{2}x + \frac{3}{4}$

$y = \frac{1}{2}x + \frac{3}{4}$



Two lines with slopes m_1 and m_2 are **perpendicular** if and only if $m_1 \cdot m_2 = -1$



$m_1 \cdot m_2 = (-3) \left(\frac{1}{3}\right) = -1$

m_2 is a negative res. of m_1

$\frac{(-3)}{-3} \cdot m_2 = \frac{-1}{-3}$

$m_2 = \frac{1}{3}$

$a \cdot \left(\frac{-1}{a}\right) = -1$

Example 1:

If you have a line with slope -2;

Any line that is **parallel** to this line has slope: -2 .

Any line that is **perpendicular** to this line has slope: $-\left(\frac{1}{-2}\right) = \frac{1}{2}$.

If you have a line with slope 7;

Any line that is **parallel** to this line has slope: 7 .

Any line that is **perpendicular** to this line has slope: $-\frac{1}{7}$.

If you have a line with slope $\frac{4}{9}$;

Any line that is **parallel** to this line has slope: $\frac{4}{9}$.

Any line that is **perpendicular** to this line has slope: $-\frac{9}{4}$.

Example 2: State whether the following lines are parallel, perpendicular, neither or the same.

$y = -5x + 4$

$y = -5x - 9$

$m_1 = -5$

$m_2 = -5$

$y = mx + b$
 $m = \text{slope}$
 $b = y\text{-int.}$

lines are parallel

Example 3: State whether the following lines are parallel, perpendicular, neither or the same.

$y = 4x + 4$

$y + \frac{1}{4}x = 2$

$-\frac{1}{4}x - \frac{1}{4}x$

$y = -\frac{1}{4}x + 2$

$m_1 = 4$

$m_2 = -\frac{1}{4}$

$4\left(-\frac{1}{4}\right) = -1$

$y = mx + b$

lines are perpendicular.

Example 4: State whether the following lines are parallel, perpendicular, neither or the same.

$3x + 2y = 6$

$-6x - 4y = -12$

$3x + 2y = 6$
 $-3x -3x$

$-6x - 4y = -12$
 $+6x +6x$

$\frac{2y}{2} = \frac{-3x+6}{2}$

$\frac{-4y}{-4} = \frac{6x-12}{-4}$

$-2(3x + 2y) = (6) - 2$

$-6x - 4y = -12$

$y = -\frac{3}{2}x + 3$

$y = -\frac{6x}{4} + 3$

$y = -\frac{3}{2}x + 3$

$m_1 = m_2$

same

Example 5: State whether the following lines are parallel, perpendicular, neither or the same.

$$10y - 5x = 15$$

$$5x + 10y = -9$$

$$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4} \neq -1$$

$$10y - 5x = 15$$

$$+5x +5x$$

$$\frac{10y}{10} = \frac{5x+15}{10}$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

$$5x + 10y = -9$$

$$-5x -5x$$

$$\frac{10y}{10} = \frac{-5x-9}{10}$$

$$y = -\frac{1}{2}x - \frac{9}{10}$$

Example 6: Write the equation of a line in slope-intercept form that passes through the point $(0, -2)$ and is parallel to the line $y = -3x + 2$.

$$m = -3$$

$$y = mx + b$$

$$y = -3x - 2$$

$(0, -2)$ is the y-int

Example 7: Write the equation of a line in slope-intercept form that passes through $(2, -6)$ and is perpendicular to the line $y - 4x = -2$.

$$y - 4x = -2$$

$$+4x +4x$$

$$y = 4x - 2$$

$$m = -\frac{1}{4}$$

Point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = -\frac{1}{4}(x - 2)$$

$$y + 6 = -\frac{1}{4}x - \frac{1}{4}(-2)$$

$$y + 6 = -\frac{1}{4}x + \frac{1}{2}$$

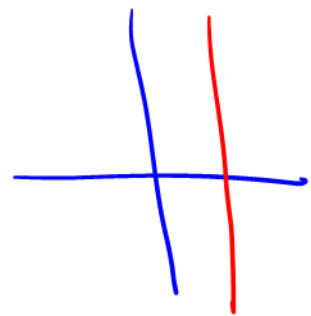
$$y = -\frac{1}{4}x + \frac{1}{2} - 6$$

$$y = -\frac{1}{4}x + \frac{1}{2} - \frac{12}{2}$$

$$y = -\frac{1}{4}x - \frac{11}{2}$$

Example 8: Find the equation of the line that passes through the point $(\frac{1}{2}, -3)$ and is perpendicular to the line $x = 4$.

vertical line
 m is undefined.
 Need a horizontal line
 $y = \text{const}$ $(\frac{1}{2}, -3)$



$$y = -\frac{1}{4}x - \frac{11}{2}$$

$$y = -3$$

$$Ax + By = C$$

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Example 9: Write the equation in **standard form** for the line that passes through the point **(2, 2)** and is **parallel** to the graph of the line $4x - 5y = -12$.

$$\begin{array}{l}
 4x - 5y = -12 \\
 -4x \qquad -4x \\
 \hline
 -5y = -4x - 12 \\
 \frac{-5y}{-5} = \frac{-4x - 12}{-5} \\
 y = \frac{4}{5}x + \frac{12}{5}
 \end{array}$$

$$\begin{array}{l}
 m = \frac{4}{5} \quad (x_1, y_1) \\
 \qquad \qquad (2, 2) \\
 y - y_1 = m(x - x_1) \\
 y - 2 = \frac{4}{5}(x - 2) \\
 y - 2 = \frac{4}{5}x - \frac{8}{5} \\
 \qquad +2 \qquad \qquad +2 \\
 y = \frac{4}{5}x + \frac{2}{5}
 \end{array}$$

$$\begin{array}{l}
 y = \frac{4}{5}x - \frac{8}{5} + \frac{2 \cdot 5}{1 \cdot 5} \\
 y = \frac{4}{5}x + \frac{-8 + 10}{5} \\
 y = \frac{4}{5}x + \frac{2}{5}
 \end{array}$$

Example 10: Write the equation of a line in **slope-intercept form** that passes through **(1, 2)** and is **parallel** to the line that passes through the points **(4, 6)** and **(6, 10)**.

$$\begin{array}{cc}
 x_1, y_1 & x_2, y_2 \\
 (4, 6) & (6, 10)
 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 6}{6 - 4} = \frac{4}{2} = 2$$

$$\begin{array}{l}
 \frac{-4}{5}x + y = \frac{2 \cdot 5}{5} \\
 \boxed{-4x + 5y = 2}
 \end{array}$$

$$\begin{array}{cc}
 m = 2 & (x_1, y_1) \\
 \text{slope} & \text{point} \\
 & (1, 2)
 \end{array}$$

Point-slope: $y - y_1 = m(x - x_1)$

$$y - 2 = 2(x - 1)$$

$$\begin{array}{l}
 y - 2 = 2x - 2 \\
 \qquad +2 \qquad \qquad +2 \\
 y = 2x
 \end{array}$$

$$\boxed{y = 2x}$$

Check: (1, 2)
 $2(1) = 2$