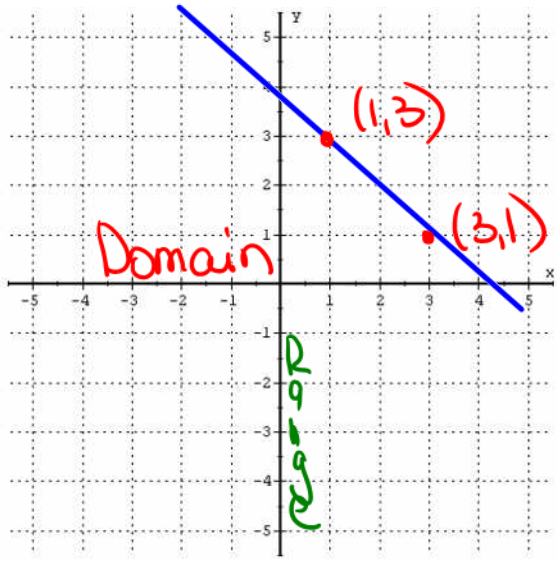
Functions and Domains:

Definition: A relation in mathematics is a set of one or more ordered pairs. It can be described by:

1. A set of ordered pairs: {(-3, -1), (-2, 1), (-1, 1), (1, 3), (3, 1), (3, 2), (0, 3)}

Domain Range Order! $(1,3) \neq (3,1)$

2. Graphs:

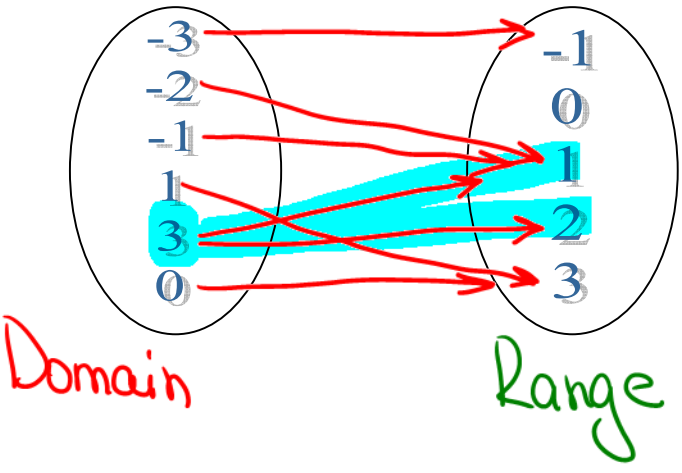


3. Tables:

Domain Range

x	y
-3	-1
-2	1
-1	1
1	3
3	1
3	2
0	3

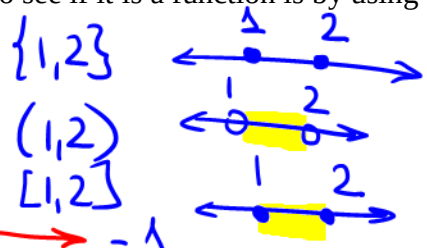
4. Mappings:



The DOMAIN of a relation is the set of all the first elements (the *x*-values or *x*-coordinates) in the ordered pairs.

The RANGE of a relation is the set of all the second elements (the *y*-values or *y*-coordinates) in the ordered pairs.

A FUNCTION is a special relation in which each element, *x*, of the domain is paired with *exactly (only) one* element, called *f*(*x*), of the range. One way to test a relation to see if it is a function is by using the vertical line test.



1. Is the given relation a function?

a) $\{(1,-1),(2,5),(3,4)\}$

Domain: $\{1,2,3\}$

Range: $\{-1,5,4\}$

function.

$1 \to -1$

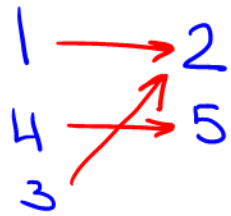
$2 \to 5$

$3 \to 4$

b) $\{(1,2),(4,5),(3,2)\}$

Domain: $\{1,4,3\}$

Range: $\{2,5\}$



function

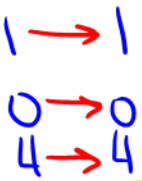
c) $\{(1,2),(4,5),(4,2)\}$

d) $\{(1,1),(0,0),(4,4)\}$

(c)

NOT a function

(d)



function

In the equation $y = f(x)$, the symbol $f(x)$ is read "f of x" and is the value of the function *f* at the number *x*. The range of *f* is the set of all possible values of $f(x)$ as *x* varies throughout the domain.

To evaluate *f* at a number, substitute the number for *x* into the definition of *f*. (Wherever there's an *x* in the equation, use the number in parentheses instead and simplify.)

2. If $f(x) = 6x - 5$, calculate $f(4)$. $= 6(4) - 5 = 24 - 5 = 19$ $f(4) = 19$

3. If $f(x) = 5x^2 + 4x - 7$, calculate $f(-2)$.

$= 5(-2)^2 + 4(-2) - 7$

$= 5(4) + 4(-2) - 7$

$= 20 - 8 - 7$

$= 5$

$f(-2) = 5$

4. If $f(x) = -2x^4 + 3x^3 + x^2 - 2x$, calculate $f(-1)$.

$f(-1) = -2(-1)^4 + 3(-1)^3 + (-1)^2 - 2(-1) = -2(1) + 3(-1) + 1 + 2$

$= -2 - 3 + 1 + 2 = -2$

$f(-1) = -2$

5. If $f(x) = \dfrac{10}{2x+4}$, calculate $f\left(\dfrac{1}{2}\right)$.

$f\left(\frac{1}{2}\right) = \dfrac{10}{2\left(\frac{1}{2}\right)+4} = \dfrac{10}{1+4} = \dfrac{10}{5} = 2$

6. If $f(x) = |2x - 5|$, calculate $f(1)$.

$f(1) = |2(1) - 5| = |2 - 5| = |-3| = 3$

$f(1) = 3$

Domain of a function:

To find the domain of a function, one must determine what all possible x-values can go into the equation to get valid y-values. We ask, "Is there anything x cannot equal?" for a function.
If the answer to the question is "no," then the domain is all real numbers, written $(-\infty, \infty)$ or $\mathbb{R}$.

If the function has an x on the bottom of a fraction, then the domain cannot contain the number(s) that makes the denominator equal to zero (since division by zero is impossible).

- fractions
- square roots

7. Find the domain of the function $f(x) = 4x + 15$.

$(-\infty, \infty)$ or $\mathbb{R}$

8. Find the domain of the function $f(x) = \dfrac{4x-1}{5}$.

$5 \neq 0$

$(-\infty, \infty)$ or $\mathbb{R}$

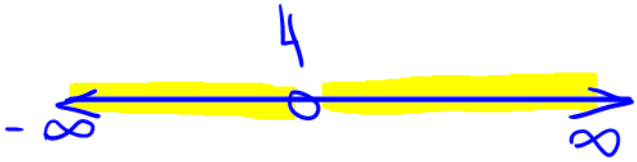
9. Find the domain of the function $f(x) = \dfrac{5}{4-x}$.

$4 - x = 0$

$-4 \quad -4$

$(-1) - x = -4(-1)$

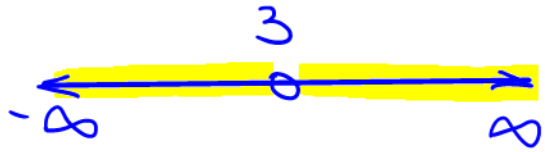
$x = 4$



$(-\infty, 4) \cup (4, \infty)$

10. Find the domain of the function $f(x)=\frac{x+4}{x-3}$.

$x-3=0$
$+3\ +3$
$x=3$

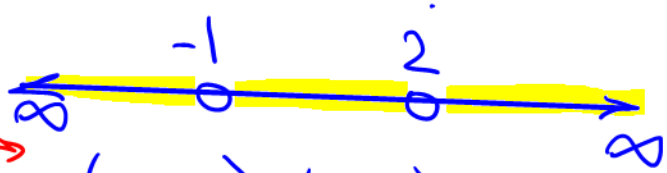


$(-\infty,3)\cup(3,\infty)$

11. Find the domain of the function $f(x)=\frac{x}{(x+1)(x-2)}$.

$(x+1)(x-2)=0$

$x+1=0$ $\quad x-2=0$
$-1\ -1$ $\quad +2\ +2$
$x=-1$ $\quad x=2$



$(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

12. Find the domain of the function $f(x)=\frac{5}{x^2-16}$.

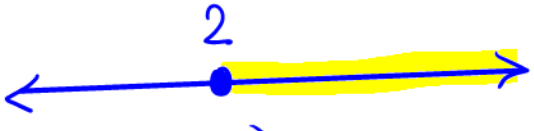
$(4)^2=16$ $\quad x^2-16=0$
$(-4)^2=16$ $\quad +16\ +16$
$\sqrt{x^2}=\sqrt{16}$ $\quad x=\pm 4$

$(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

Radical Functions: If the function has an x on the inside of a square root sign, then the domain cannot contain those numbers which make the inside negative (since we can't take the square root of a negative number).

13. Find the domain of the function $f(x)=\sqrt{x-2}$.

$x-2\geq 0$
$+2\ +2$
$x\geq 2$

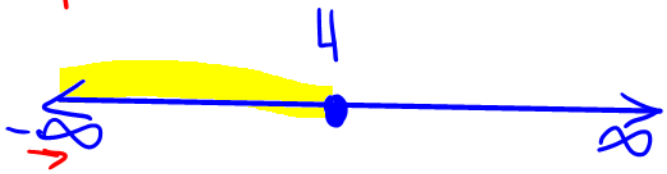


$[2,\infty)$

14. Find the domain of the function $f(x)=\sqrt{8-2x}$.

$8-2x\geq 0$
$-8\ \ -8$
$-2x\geq -8$ $\quad$ flip
$\frac{-2x}{-2}\geq\frac{-8}{-2}$

$x\leq 4$



$(-\infty,4]$

$f=\frac{5}{\sqrt{x-1}}$