$\sqrt{x} = x^{1/2}$ $\frac{1}{x} = x^{-1}$

Polynomials

A polynomial is an expression made up of terms called monomials.

A monomial is an expression made up of one real-number coefficient and (at least) one variable to some **whole-number** power.

Examples are -3; $7x$; $9x^2y^3z^5$.

A polynomial made up of two monomials added to or subtracted from each other is called a binomial.

Examples: $3xy + 6z^2$; $2x - 8x^3$; $4abc^2 - ab^3$.

A polynomial made up of three monomials added to or subtracted from each other is called a trinomial.

Examples: $4abc^2 - ab^2 + 5c$; $3x^2 + 9x^6 - 6$.

(Generally we do not name polynomials with four or more terms.)

Usually, polynomials that use only one variable are written with the highest exponent first and then in descending order of exponents: $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$

The **degree of a monomial** is the sum of the powers of the variables. Do not use the coefficient when determining the degree.

Examples:	$3x^2$	$-6x^3y^2$	$2x^1y^1z^1$	$16x^0$
Degree?	2	$3+2=5$	$1+1+1=3$	0

The **degree of a polynomial** is determined by the degrees of the monomials. Find the degree of each monomial (each term), the highest degree is the degree of the polynomial.

Example: $5x^2 + 4x + 10$; the degree is 2.

Term:	$5x^2$	$4x$	10
Degree:	2	1	0

$x^2y^3 + 4xy^8 + 6y^7$; the degree is 9.

Term:	x^2y^3	$4x^1y^8$	$6y^7$
Degree:	5	9	7

A zero-degree polynomial is just a number without a variable (also called a constant). Example: $9x^0$

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A first-degree polynomial is a linear function. Examples: $2x^1 + 3$; $\frac{2}{3}x^1 - 5$.

A second-degree polynomial is called a quadratic function. Example: $x^2 + 2x - 1$.

A third-degree polynomial is called a cubic function. Example: $x^3 + 5x^2 + 4$

The leading term of a polynomial is the monomial with the largest exponent or of the highest degree.

Examples: $\boxed{5x^6} + 10x^4 + 1$; the leading term is $5x^6$.
$4x^2y^3 \boxed{- 6x y^{1} {}^{8}}$; the leading term is $-6xy^8$.

Degree: 5 9

The leading coefficient of a polynomial is the coefficient of the monomial with the largest exponent or of the highest degree.

Examples: $5x^6 + 10x^4 + 1$; the leading coefficient is 5.
$4x^2y^3 - 6xy^8$; the leading coefficient is -6.

Like terms are two monomials that are the same, or differ only by their numerical coefficients.

Examples:	$8x^2$ and $-5x^2$	$3x^2y$ and $-9xy^2$	$2x^3$ and $2x^4$
Like or unlike?	like	unlike	unlike

In the following examples, is each expression a polynomial? If yes: what is the degree of the polynomial, what type of polynomial is it (monomial, binomial, trinomial, or none of these), what is the leading coefficient, and what is the constant term?

1) $x^2 - 3x + 1$ Poly., Deg. = 2, trinomial, LC = 1, CT = 1

2) $\frac{7}{2}x^4 - \frac{5}{3}x^3 + 9$ Poly., Deg. = 4, trinomial, LC = $\frac{7}{2}$, CT = 9

3) $10x^{-2} + 4x^{-1} + 5$ Not a poly

4) $3a^3b^4 - 2a^2b^2$ Poly, Deg = 7, Binomial, LC = 3, CT = 0

7 4

5) $-a^7 + 2a^3b^5 + b^5 - 3a^2b^4$ Poly, Deg = 8, LC = 2, CT = 0

7 8 5 6

6) $\sqrt{x} + x^3 - 5$ $= x^{1/2} + x^3 - 5$ Not a Poly.

7) $7x - 2x^3$ Poly, Deg = 3, binomial, LC = -2, CT = 0

1 3

8) $3x^7 - 2x^4y^6 - 3y^8$ Poly, Deg = 10, trinomial, LC = -2, CT = 0

7 10 8

9) $\dfrac{1}{x} + x^2 + 4$ Not a Poly.

$\sqrt{2}x^2 + 5x - 125$ Poly

10) $\dfrac{2x^3 + x}{x^2 + 5}$ Not a Poly

The **domain** of a polynomial function is $(-\infty, \infty)$. Any real number can be put into the function in place of the variable.

To evaluate the polynomial function $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ at a number $x = b$ (or to evaluate $f(b)$), take the number b and place it in the function everywhere an x appears in the original function.

Examples:

Evaluate $f(x) = 3x + 7$ at $x = 3$.

$f(3) = 3(3) + 7$	Put (3) everywhere there's an x in the original function.
$f(3) = 9 + 7$	Simplify
$f(3) = 16$	Answer

Given the function $f(x) = 2x^3 - 7x^2 + 5$, find $f(-2)$.

$f(-2) = 2(-2)^3 - 7(-2)^2 + 5$	Replace all the x's with (-2)'s.
$f(-2) = 2(-8) - 7(4) + 5$	Simplify
$f(-2) = -16 - 28 + 5$	Simplify more
$f(-2) = -39$	Answer

Evaluate $f(1)$ if $f(x) = \boxed{19}$. $f(x)=y$

$f(1) = 19$ Replace all x's with (1). This is the answer; there are no x's in the original equation.

Example 11: Evaluate $7x - 2x^3$ at $x = 2$.

$$7(2)-2(2)^3 = 7(2)-2(8) = 14-16 = \boxed{-2}$$

Example 12: Given $f(x) = 2x + 3$, find $f(0)$.

$$f(0) = 2(0)+3 = 3 \qquad \boxed{f(0)=3}$$

Example 13: Evaluate $f(2)$ if $f(x) = x^3 + 1$.

$x=2$

$$f(2) = (2)^3+1 = 8+1 = 9 \qquad \boxed{f(2)=9}$$

Example 14: If $f(x) = x^3 - 2x^2 + 5x + 4$, find $f(-1)$.

$$f(-1) = (-1)^3 - 2(-1)^2 + 5(-1) + 4$$
$$= -1 - 2(1) - 5 + 4$$
$$= -1 - 2 - 5 + 4$$
$$= -4$$

$$\boxed{f(-1) = -4}$$