

$$3, 5$$

$$3x, 5x$$

$$3x, 5x^2 \leftarrow \text{not like}$$

$$3xy^2, 5x^2y$$

Popper #8 5As

Operations with Polynomials

Addition of Polynomials

The **sum** of two polynomials is found by **combining like terms**. To **add like terms**, **add the coefficients** and **do not change the variable and exponents** in common.

Examples: $4x + 5x = 9x$, $2x^2 + 5x^2 = 7x^2$

Example: Perform the addition $(2x^7 + 9x^3 - 5) + (3x^3 + 2x + 14)$.

$$\begin{aligned} (2x^7 + 9x^3 - 5) + (3x^3 + 2x + 14) & \text{ Remove the parentheses and group like terms together} \\ 2x^7 + 9x^3 + 3x^3 + 2x + 14 - 5 & \text{ Combine like terms. Don't change variables or exponents.} \\ 2x^7 + 12x^3 + 2x + 9 & \text{ Answer.} \end{aligned}$$

Example: Perform the addition $(2x^3 - 17x^2 - 5x) + (3x^3 - x + 8)$.

$$\begin{aligned} (2x^3 - 17x^2 - 5x) + (3x^3 - x + 8) & \text{ Remove the parentheses and group like terms together} \\ 2x^3 + 3x^3 - 17x^2 - 5x - x + 8 & \text{ Combine like terms. Don't change variables or exponents.} \\ 5x^3 - 17x^2 - 6x + 8 & \text{ Answer.} \end{aligned}$$

Subtraction of Polynomials

The **difference** of two polynomials is found by **adding the first polynomial** to the **negative of the second polynomial**. The **negative** of the second polynomial is found by **changing the sign of each term** of the polynomial.

Example: Perform the subtraction $(9x^5 + 2x^3 - 1) - (2x^3 + 4x - 4)$.

$$\begin{aligned} (9x^5 + 2x^3 - 1) - (2x^3 + 4x - 4) & \text{ Create the negative of the second polynomial.} \\ (9x^5 + 2x^3 - 1) + (-2x^3 - 4x + 4) & \text{ Add, removing the parentheses and grouping like terms.} \\ 9x^5 + 2x^3 - 2x^3 - 4x - 1 + 4 & \text{ Combine like terms.} \\ 9x^5 - 4x + 3 & \text{ Answer.} \end{aligned}$$

Example: Perform the subtraction $(5x^2 - 7x + 2) - (x^2 + 4x - 3)$.

$$\begin{aligned} (5x^2 - 7x + 2) - (x^2 + 4x - 3) & \text{ Create the negative of the second polynomial.} \\ (5x^2 - 7x + 2) + (-x^2 - 4x + 3) & \text{ Add, removing the parentheses and grouping like terms.} \\ 5x^2 - x^2 - 7x - 4x + 2 + 3 & \text{ Combine like terms.} \\ 4x^2 - 11x + 5 & \text{ Answer.} \end{aligned}$$

Perform the indicated operations. Write your final answer with the terms in descending order, from highest to lowest degree.

$$1) (2x - 4) + (5x + 6) = \underline{2x - 4} + \underline{5x + 6} = \boxed{7x + 2}$$

$$2) (4x - 1) - (6x - 7) = \underline{4x - 1} - \underline{6x - 7} = \boxed{-2x + 6}$$

$$(4x - 1) + (-6x + 7)$$

$$3) (4x^4 - x^2) - (2x^4 - 5x + 4) = \underline{4x^4 - x^2} - \underline{2x^4 + 5x - 4} = \boxed{2x^4 - x^2 + 5x - 4}$$

$$(4x^4 - x^2) + (-2x^4 + 5x - 4)$$

$$4) 5x^2 - (x^4 - 3x^2) = 5x^2 + \underline{-x^4 + 3x^2} = \underline{5x^2 - x^4} + \underline{3x^2}$$

$$= 8x^2 - x^4 = \boxed{-x^4 + 8x^2}$$

$$5) (3x^3 - 4x^2 + 2) - (4x^3 + x^2 + 2x - 6) = \underline{(3x^3 - 4x^2 + 2)} + \underline{(-4x^3 - x^2 - 2x + 6)}$$

$$= \underline{3x^3 - 4x^2 + 2} - \underline{4x^3 - x^2 - 2x + 6}$$

$$6) (3x^2 - 2x) + (2x - 6) = \underline{3x^2} - \underline{2x} + \underline{2x} - \underline{6} = \boxed{3x^2 - 6}$$

$$= \underline{-x^3 - 5x^2 - 2x + 8}$$

$$7) (3x^2 + 2x - 1) + (2x^2 - 5x + 4) = \underline{3x^2 + 2x - 1} + \underline{2x^2 - 5x + 4}$$

$$= \boxed{5x^2 - 3x + 3}$$

$$8) (3x^2 + 2x - 1) - (2x^2 - 5x + 4) = \underline{3x^2 + 2x - 1} - \underline{2x^2 - 5x + 4}$$

$$= \underline{x^2 + 7x - 5}$$

$$9) (x^4 + 2x - 2) - (2x^3 - 5x^2 + 1)$$

$$= \underline{(x^4 + 2x - 2)} + \underline{(-2x^3 + 5x^2 - 1)}$$

$$= \underline{x^4 + 2x - 2} - \underline{2x^3 + 5x^2 - 1}$$

$$= \boxed{x^4 - 2x^3 + 5x^2 + 2x - 3}$$

$$x^n (x^m) = x^{n+m}$$

Multiplication of Polynomials

To multiply two monomials, multiply the coefficients and multiply the corresponding variables.

Examples: $x \cdot x = x^2$, $(2x)(5x) = 10x^2$, $(4x^3)(x^5) = 4x^8$, $(2xy)(7x^2y) = 14x^3y^2$

Example: $(4x^2)(3x^6) = 12x^{2+6} = 12x^8$ $x^3(x^5) = x^{3+5} = x^8$

Example: $(2x^2y)(5x^4y^3) = 10x^{2+4}y^{1+3} = 10x^6y^4$

Multiplication of polynomials is done by repeated use of the distributive property. Multiplication of **binomials** (two-termed polynomials) is done using the **FOIL** method. FOIL is an acronym which stands for “**first terms, outside terms, inside terms, last terms.**”

Example: Perform the multiplication $(-7x + 2)(4x - 3)$.

$$\begin{aligned} &(-7x + 2)(4x - 3) \\ &(-7x)(4x) + (-7x)(-3) + 2(4x) + 2(-3) \\ &-28x^2 + 21x + 8x - 6 \\ &-28x^2 + 29x - 6 \end{aligned}$$

Multiply the first, outside, inside and last terms and add.
Simplify.
Combine like terms.
Answer.

Example: Perform the multiplication $(9x^3 - 5)(3x^3 + 2x)$.

$$\begin{aligned} &(9x^3 - 5)(3x^3 + 2x) \\ &(9x^3)(3x^3) + (9x^3)(2x) + (-5)(3x^3) + (-5)(2x) \\ &27x^6 + 18x^4 - 15x^3 - 10x \end{aligned}$$

Multiply the first, outside, inside and last terms and add.
Simplify.
Answer. There are no like terms to combine.

Example: Perform the multiplication $(2x^3 + 9x^2 - 5)(3x^2 + 2x + 14)$.

$$\begin{aligned} &(2x^3 + 9x^2 - 5)(3x^2 + 2x + 14) \\ &(2x^3)(3x^2) + (2x^3)(2x) + (2x^3)(14) + (9x^2)(3x^2) + (9x^2)(2x) + (9x^2)(14) + (-5)(3x^2) + (-5)(2x) + (-5)(14) \\ &6x^5 + 4x^4 + 28x^3 + 27x^4 + 18x^3 + 126x^2 - 15x^2 - 10x - 70 \\ &6x^5 + 4x^4 + 27x^4 + 28x^3 + 18x^3 + 126x^2 - 15x^2 - 10x - 70 \\ &6x^5 + 31x^4 + 46x^3 + 111x^2 - 10x - 70 \end{aligned}$$

Distribute each term in the first group with each term in the second and add the results together.
Multiply.
Group like terms together.
Combine like terms.
Answer.

Multiply. Write your final answer with the terms in descending order, from highest to lowest degree.

$$1) (2x)(5x) = 10x^2$$

$$2) 2x(4x+3) = 2x(4x) + 2x(3) = 8x^2 + 6x$$

$$3) (x-4)(x+2) = x(x) + x(2) + (-4)(x) + (-4)(2) \\ = x^2 + \underline{2x} - \underline{4x} - 8 = x^2 - 2x - 8$$

$$4) (2x-5)(3x-2) = (2x)(3x) + (2x)(-2) + (-5)(3x) + (-5)(-2) \\ = 6x^2 - \underline{4x} - \underline{15x} + 10 = 6x^2 - 19x + 10$$

$$5) (x+5)^2 = (x+5)(x+5) = x(x) + x(5) + 5(x) + 5(5) \\ = x^2 + \underline{5x} + \underline{5x} + 25 = x^2 + 10x + 25$$

$$6) (x-6)^2 = (x-6)(x-6) = x(x) + x(-6) + (-6)(x) + (-6)(-6) \\ = x^2 - \underline{6x} - \underline{6x} + 36 = x^2 - 12x + 36$$

$$7) (x^2-5)(2x^2+4) = x^2(2x^2) + x^2(4) + (-5)(2x^2) + (-5)(4) \\ = 2x^4 + 4x^2 - 10x^2 - 20 = 2x^4 - 6x^2 - 20$$

$$8) (x-5)(2x^2+x+4) = 2x^3 + \underline{x^2} + \underline{4x} - \underline{10x^2} - \underline{5x} - 20 = 2x^3 - 9x^2 - x - 20$$

$$9) (2x-5)^2 = (2x-5)(2x-5) \\ = 4x^2 - \underline{10x} - \underline{10x} + 25 \\ = 4x^2 - 20x + 25$$

Perform the indicated operations. Write your final answer with the terms in descending order, from highest to lowest degree.

$$1) 2x^7 6x^4 5x^3 = 60x^{7+4+3} = \boxed{60x^{14}}$$

$$2) -5x(2x^2 - 4x + 10) = (-5x)(2x^2) + (-5x)(-4x) + (-5x)(10) \\ = \boxed{-10x^3 + 20x^2 - 50x}$$

$$3) (x+4)(x-1) \\ = x^2 - x + 4x - 4 = \boxed{x^2 + 3x - 4}$$

$$4) (x+2y)(x^2 - xy + y^2) = x^3 + x(-xy) + xy^2 + 2x^2y + 2y(-xy) + 2y^3 \\ = x^3 - xy^2 + xy^2 + 2x^2y - 2xy^2 + 2y^3 \\ = \boxed{x^3 + x^2y - xy^2 + 2y^3}$$

$$5) 2y - 5y(y^2 + 4) \\ = \underline{2y} - 5y^3 - 20y = \boxed{-5y^3 - 18y}$$

$$6) (x+1)(5x+7) = x(5x) + x(7) + 1(5x) + 1(7) \\ = 5x^2 + 7x + 5x + 7 = \boxed{5x^2 + 12x + 7}$$

$$7) (2x^2y^3)(4xy^2) = \boxed{8x^3y^5}$$

$$8) (2xy^4)(5x^2z^2) = \boxed{10x^3y^4z^2}$$

Ex.

$$(x-5)(x+2)^2$$

$$= \underline{(x-5)(x+2)}(x+2)$$

$$= \underline{(x^2 + 2x - 5x - 10)}(x+2)$$

$$= (x^2 - 3x - 10)(x+2)$$

$$= x^3 + \underline{2x^2} - 3x^2 - \underline{6x} - \underline{10x} - 20$$

$$= \boxed{x^3 - x^2 - 16x - 20}$$