Greatest Common Factor and Factoring by Grouping

$a \cdot b$

(Review) Factoring

Definition: A factor is a number, variable, monomial, or polynomial which is multiplied by another number, variable, monomial, or polynomial to obtain a product.

1. List all the possible factors of the following numbers:

a. 12 1, 2, 3, 4, 6, 12

b. 32 1, 2, 4, 8, 16, 32

c. 19 1, 19

d. 45 1, 3, 5, 9, 15, 45

In the above, the number 19 is an example of a __prime__ number because its only positive factors are one and itself.

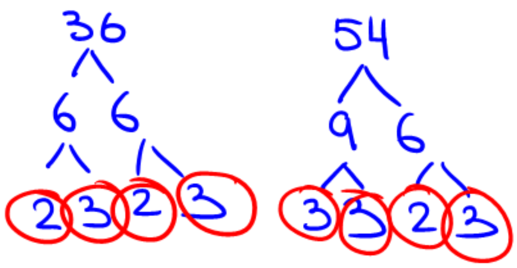
(Review) Greatest Common Factor

Definition: The **greatest common factor** of two or more numbers is the largest number that divides (goes into) the given numbers with a remainder of zero.

2. Find the GCF (greatest common factor) of the following numbers:

a. 36 and 54

$GCF = 2 \cdot 3^2 = \boxed{18}$

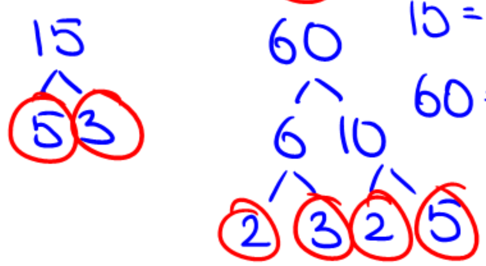


$36 = 2^2 \cdot 3^2$

$54 = 2 \cdot 3^3$

b. 15 and 60

$GCF = 3(5) = 15$



$15 = 3 \cdot 5$

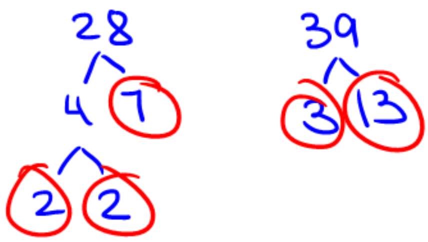
$60 = 2^2 \cdot 3 \cdot 5$

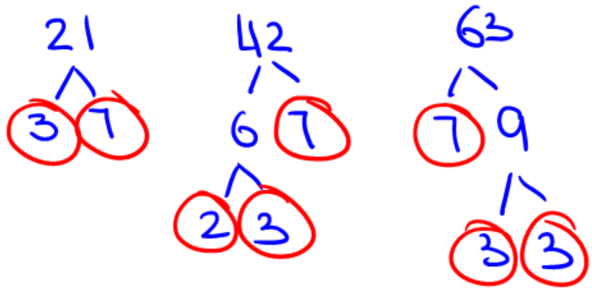
c. 21, 42, and 63

$GCF = 3(7) = 21$

d. 28 and 39 relativly prime

$GCF = 1$





$21 = 3 \cdot 7$

$42 = 2 \cdot 3 \cdot 7$

$63 = 3^2 \cdot 7$

Greatest Common Factor of Polynomials
In order to find the GCF of two or more monomials,
I. Find the GCF of the coefficients;
II. Find the GCF of the variables;
III. Rewrite the GCF as a product of the GCF of the coefficients times the GCF of the variables.

Examples:

1. Find the GCF of x^4 and x^7.
   Step I: The only coefficients are 1's, so this is the GCF of the coefficients.
   Step II: Rewrite the two monomials as products of $\underline{x}$s without using exponents:
   $x^4 = x \cdot x \cdot x \cdot x$
   $x^7 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$
   Since each monomial has 4 $\underline{x}$s in it, the GCF of the variables is x^4.
   Step III: The GCF of x^4 and x^7 is x^4.

2. Find the GCF of xy^2 and x^5y^4.
   Step I: The only coefficients are 1's, so this is the GCF of the coefficients.
   Step II: Rewrite the two monomials as products of $\underline{x}$s and $\underline{y}$s without using exponents:
   $xy^2 = x \cdot y \cdot y$
   $x^5y^4 = x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
   Since each monomial has 1 $\underline{x}$ and 2 $\underline{y}$s in it, the GCF of the variables is xy^2.
   Step III: The GCF of xy^2 and x^5y^4 is xy^2.

3. Find the GCF of $24x^4y$ and $9x^7y^4$.
   Step I: The GCF of the coefficients 24 and 9 is 3.
   Step II: Rewrite the two monomials as products of $\underline{x}$s and $\underline{y}$s without using exponents:
   $x^4y = x \cdot x \cdot x \cdot x \cdot y$
   $x^7y^4 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
   Since each monomial has 4 $\underline{x}$s and 1 $\underline{y}$ in it, the GCF of the variables is x^4y.
   Step III: The GCF of $24x^4y$ and $9x^7y^4$ is $3x^4y$.

Examples:

1. Find the GCF of $18x$ and $27x^3$.

GCF = $\boxed{9x}$

2. Find the GCF of $5x^4$ and $9x^2$.

GCF = $\boxed{x^2}$

3. Find the GCF of $10x^5$ and $50x$.

GCF = $\boxed{10x}$

4. Find the GCF of $14xy$ and $21x^5y^2$.

GCF = $\boxed{7xy}$

14 → 2, 7

21 → 3, 7

14 = 2 · 7

21 = 3 · 7

5 Find the GCF of $8x^9$ and $6x^4y$.

GCF = $\boxed{2x^4}$

6. Find the GCF of $45a^6b^3c$ and $60a^3bc^2$.

GCF = $\boxed{15a^3bc}$

7. Find the GCF of $8x$, $16x^3$ and $32x^6$.

GCF = $\boxed{8x}$

8. Find the GCF of $9x^5$, $16x^3$ and $5x^7$.

GCF = $\boxed{x^3}$

9. Find the GCF of $8xy^2$, $2x^3y$ and $10x^6y^3$.

GCF = $\boxed{2xy}$

10. Find the GCF of $24a^2b$, $21a^4b^3$ and $12a^4b^6$.

GCF = $\boxed{3a^2b}$

Factoring

To factor a polynomial, try to find the GCF of the monomials in the polynomial. Then this GCF should be factored out of the polynomial by "undistributing" the GCF out of all the monomials in the polynomial. Note that if the leading coefficient is negative, then the GCF should also be negative.

Examples:

1. Factor $7x + 14y$.
   Step I: Find the GCF of the coefficients. GCF(7, 14) = 7.
   Step II: Find the GCF of the variable parts. There is none, so nothing changes.
   Step III: Divide all the monomials by the GCFs and rewrite with the GCFs out front:

$$7x + 14y = 7\left(\frac{\cancel{7}x}{\cancel{7}} + \frac{\overset{2}{\cancel{14}}y}{\cancel{7}}\right) = 7(x + 2y)$$

$7x + 14y$

2. Factor $8x^3 + 6x^2$.
   Step I: Find the GCF of the coefficients. GCF(8, 6) = 2.
   Step II: Find the GCF of the variable parts. GCF(x^3, x^2) = x^2.
   Step III: Divide all the monomials by the GCFs and rewrite with the GCFs out front:

$$8x^3 + 6x^2 = 2x^2\left(\frac{8x^3}{2x^2} + \frac{6x^2}{2x^2}\right) = 2x^2(4x + 3)$$

$8x^3 + 6x^2$

$\frac{\overset{4}{\cancel{8}}x^3}{\cancel{2}x^2} = 4x^{3-2} = 4x$

$\frac{\overset{3}{\cancel{6}}\cancel{x^2}}{\cancel{2}\cancel{x^2}} = 3$

Examples:

1. Factor $5x^2 + 7x$ $= x\left(\frac{5x^2}{x} + \frac{7x}{x}\right)$ $= \boxed{x(5x+7)}$

2. Factor $8x^2 + 4x$ $= 4x\left(\frac{8x^2}{4x} + \frac{4x}{4x}\right) = \boxed{4x(2x+1)}$

3. Factor $12x^3 - 6x^2$ $= \boxed{6x^2(2x-1)}$

4. Factor $16x^4 + 8x^6$ $= 8x^4\left(\frac{\overset{2}{\cancel{16}}\cancel{x^4}}{\cancel{8}\cancel{x^4}} + \frac{\cancel{8}x^6}{\cancel{8}x^4}\right) = \boxed{8x^4(2+x^2)}$

5. Factor $12x^3y + 2x^2y^2$ $= 2x^2y\left(\frac{\overset{6}{\cancel{12}}\overset{x}{\cancel{x^3}}\cancel{y}}{\cancel{2}\cancel{x^2}\cancel{y}} + \frac{\cancel{2}\cancel{x^2}\overset{y}{\cancel{y^2}}}{\cancel{2}\cancel{x^2}\cancel{y}}\right)$

$= \boxed{2x^2y(6x+y)}$

6. Factor $5ab+6a$. $= \boxed{a(5b+6)}$

7. Factor $8a^2bc-12ab^2c$. $= \boxed{4abc(2a-3b)}$

8. Factor $-28a^3b^7-36a^2b^5$. $= \boxed{-4a^2b^5(7ab^2+9)}$

9. Factor $12x^4y^6+4xy^3-16x^2y^5$ $= \boxed{4xy^3(3x^3y^3+1-4xy^2)}$

10. Factor $-12x^5+4x^3-8x^2$ $= \boxed{-4x^2(3x^3-x+2)}$

11. Factor $10a^2b^6+6ab^3c-8a^2b^5c^2$ $= \boxed{2ab^3(5ab^3+3c-4ab^2c^2)}$

12. Factor $y(x-1)+6(x-1)$ $= \boxed{(x-1)(y+6)}$

13. Factor $x(y+2)-z(y+2)$ $= \boxed{(y+2)(x-z)}$

14. Factor $2x(y-2)+6x^2(y-2)$ $= \boxed{2x(y-2)(1+3x)}$

Factoring by Grouping

If a polynomial contains four or more terms, it may be helpful to put the terms into groups of two and factor out a common factor from each of these groups. This is called grouping.

Examples:

1. Factor $x^2y + 6x + 3xy^2 + 18y$

Step I: Group the terms so that each group shares a common factor:
$x^2y + 6x + 3xy^2 + 18y = (x^2y + 6x) + (3xy^2 + 18y)$

Step II: Factor out the common terms from each group:
$(x^2y + 6x) = x(xy + 6)$
$(3xy^2 + 18y) = 3y(xy + 6)$

Step III: Rewrite the polynomial as the sum of the factored groups:
$x^2y + 6x + 3xy^2 + 18y = x(xy + 6) + 3y(xy + 6)$

Step IV: Factor the resulting polynomial from Step III:
$x(xy + 6) + 3y(xy + 6) = (xy + 6)(x + 3y)$

2. Factor $2x^3 + 3x^2 + 2x + 3$

Step I: Group the terms so that each group shares a common factor:
$2x^3 + 3x^2 + 2x + 3 = (2x^3 + 3x^2) + (2x + 3)$

Step II: Factor out the common terms from each group:
$(2x^3 + 3x^2) = x^2(2x + 3)$
$(2x + 3) = 1(2x + 3)$ (Note: There's no common term, so the GCD is 1.)

Step III: Rewrite the polynomial as the sum of the factored groups from Step II.
$2x^3 + 3x^2 + 2x + 3 = x^2(2x + 3) + 1(2x + 3)$

Step IV: Factor out the resulting polynomial from Step III:
$x^2(2x + 3) + 1(2x + 3) = (x^2 + 1)(2x + 3)$

3. Factor $3x^3 + 3x^2 - 4x - 4.$

Step I: Group the terms so that each group shares a common factor:
$3x^3 + 3x^2 - 4x - 4 = (3x^3 + 3x^2) + (-4x - 4)$

Step II: Factor out the common terms from each group:
$(3x^3 + 3x^2) = 3x^2(x + 1)$
$(-4x - 4) = -4(x + 1)$

Step III: Rewrite the polynomial as the sum of the factored groups from Step II.
$3x^3 + 3x^2 - 4x - 4 = 3x^2(x + 1) + (-4(x + 1)) = 3x^2(x + 1) - 4(x + 1)$

Step IV: Factor out the resulting polynomial from Step III:
$3x^2(x + 1) - 4(x + 1) = (3x^2 - 4)(x + 1)$

Examples:

1. Factor $4a^3 + 8a + 3a^2b^2 + 6b^2$.

$$= 4a(a^2+2) + 3b^2(a^2+2)$$
$$= (a^2+2)(4a+3b^2)$$

2. Factor $3a^2x + a^2y - 12x - 4y$.

$$= a^2(3x+y) - 4(3x+y)$$
$$= (3x+y)(a^2-4) = (3x+y)(a-2)(a+2)$$

3. Factor $6ax + 6xb + 10ay + 10yb$.

$$6x(a+b) + 10y(a+b)$$
$$(a+b)(6x+10y) = 2(a+b)(3x+5y)$$

4. Factor $6x + 9xy + 10y + 15y^2$.

$$= 3x(2+3y) + 5y(2+3y)$$
$$= (2+3y)(3x+5y)$$