Factoring Polynomials

Some trinomials that can be factored do not look like the special trinomials from the previous sections. Factor trinomials, written $ax^2 + bx + c$, by doing the following rules:

1. Factor out the GCF of all three terms. Use the resulting trinomial for the rest of the steps. If a is negative, also factor out -1 along with the GCF.
2. Check that the square root of $b^2 - 4ac$ is a whole number (that is $b^2 - 4ac$ is a perfect square) . If $b^2 - 4ac$ is negative, then we cannot factor the trinomial. If the square root of $b^2 - 4ac$ is not a whole number, then the factored form of $ax^2 + bx + c$ will have fractions or square root signs in it. We will not be factoring these in this section.
3. Look at the sign of the constant term.
   a. If the second sign (the one before the constant term) is a + sign, then both signs in the factored form are whatever the first sign is.
      $ax^2 + bx + c = (\_\_ + \_\_)(\_\_ + \_\_)$
      or
      $ax^2 - bx + c = (\_\_ - \_\_)(\_\_ - \_\_)$
   b. If the second sign is a − sign, then the signs in the factored form are different.
      $ax^2 + bx - c = (\_\_ + \_\_)(\_\_ - \_\_)$
      or
      ~~$ax^2 + bx - c = (\_\_ + \_\_)(\_\_ - \_\_)$~~
4. Find two numbers that multiply together to give ac but add up to give b. Keep the signs of a, b, and c with the numbers.
5. Rewrite in preliminary factored form:
   a. Write the GCF on the outside of the factored form.
   b. In both sets of the parentheses, write ax
   c. Write the appropriate signs from step 3.
   d. Write the two numbers found in step 4, one in each set of parentheses.
6. Factor out any GCFs from the sets of parentheses and throw them away.
7. Rewrite. This is the final factored form.

Examples:

1. Factor $x^2 + 5x + 6$.

   Step 1: Find GCF(x^2, $5x$, 6). The GCF of this trinomial is 1 since the only factor of 1 is 1.

   Step 2: Find $b^2 - 4ac$. Here, a = 1, b = 5, and c = 6, so $b^2 - 4ac = (5)^2 - (4)(1)(6) = 25 - 24 = 1$. Since $\sqrt{1} = 1$, a whole number, then we can factor this trinomial and have integer coefficients.

   Step 3: Sign of the constant term. The constant term of this trinomial is +6, so both the signs inside the factored form (a.k.a. the answer) will be the same. Since the sign on the $5x$ is positive also, the factored form will look like $(\_\_ + \_\_)(\_\_ + \_\_)$.

Step 4: Find 2 numbers. We need two numbers that multiply together to give (1)(6) = 6 and add up together to give 5. Factors of 6 are 1 & 6 or 2 & 3. Since 2 + 3 = 5, these are the numbers we will use.
Step 5: Preliminary answer. The preliminary factored form is $1(1x + 2)(1x + 3)$.
Step 6: Internal GCFs. There are no GCFs other than 1 inside each set of parentheses.
Step 7: Rewrite. The factored form is $(x + 2)(x + 3)$

2. Factor the polynomial $x^2 - 5x - 24$.
Step 1: GCF(x^2, $-5x$, -24) = 1.
Step 2: $b^2 - 4ac = (-5)^2 - 4(1)(-24) = 25 + 96 = 121$. $\sqrt{121} = 11$, so we can factor this polynomial.
Step 3: The sign of the constant term is negative, so the signs inside the sets of parentheses will be different, (__ + __)(__ − __)
Step 4: We want 2 numbers that multiply together to give (1)(-24) = -24 and add up to -5. Factors of -24 and their sums are

1st	2nd	Product	Sum
-1	24	-24	23
-2	12	-24	10
-3	8	-24	5
-4	6	-24	2

→

1st	2nd	Product	Sum
1	-24	-24	-23
2	-12	-24	-10
3	-8	-24	-5
4	-6	-24	-2

The numbers 3 and -8 multiply together to give -24 and add up to -5.
Step 5: The preliminary answer is $1(1x + 3)(1x - 8)$.
Step 6: Neither set of parentheses have a GCF other than 1.
Step 7: The answer is $(x + 3)(x - 8)$.

3. Factor $x^2 + 10x + 18$.
Step 1: GCF(x^2, $10x$, 18) = 1.
Step 2: $b^2 - 4ac = (10)^2 - (4)(1)(18) = 100 - 72 = 28$. Since 28 is not a perfect square (the square root of 28 is not a whole number), we cannot factor this trinomial and have only integer coefficients in our answer. This trinomial is in its final factored form.

4. Factor $3x^2 + 5x + 4$.
Step 1: Find GCF($3x^2$, $5x$, 4). The GCF is 1 since none of the coefficients have a factor in common.
Step 2: Find $b^2 - 4ac$. In this trinomial, $a = 3$, $b = 5$, and $c = 4$, so $b^2 - 4ac = (5)^2 - (4)(3)(4) = 25 - 48 = -23$. Since -23 is negative, we cannot factor this polynomial.

$ax^2 + bx + c$

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$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

Example 1: Factor $1x^2 - 5x + 6$

$\boxed{b^2 - 4ac} = (-5)^2 - 4(1)(6)$

$a = 1$

$b = -5$

$c = 6$

$= 25 - 24 = 1$

1 is a perfect square $(1)^2 = 1$

$x^2 - 5x + 6 = (x-3)(x-2)$

Find 2 factors of 6 that add up to -5

-3, -2

Example 2: Factor $x^2 - 4x - 5$ $= \boxed{(x-5)(x+1)}$

$b^2 - 4ac = (-4)^2 - 4(1)(-5)$

$a = 1$

$b = -4$

$c = -5$

$= 16 - (-20)$

$= 36$

Find 2 factors of -5 that add up to -4

-5, 1

Example 3: Factor $x^2 - 11x + 10$ $= \boxed{(x-10)(x-1)}$

$b^2 - 4ac = (-11)^2 - 4(1)(10)$

$a = 1$

$b = -11$

$c = 10$

$= 121 - 40$

$= 81$

Find 2 factors of 10 that add up to -11

-10, -1

Example 4: Factor $2x^2 + 18x - 20$ $= 2(x^2 + 9x - 10) = \boxed{2(x+10)(x-1)}$

$b^2 - 4ac = (9)^2 - 4(1)(-10)$

$a = 1$

$b = 9$

$c = -10$

$= 81 - (-40)$

$= 81 + 40$

$= 121 = (11)^2$

$x^2 + 9x - 10 = (x+10)(x-1)$

Find 2 factors of -10 that add up to 9

10, -1

Example 5: Factor $6x^2 - 18x + 12$ $= 6(x^2 - 3x + 2) = \boxed{6(x-2)(x-1)}$

$b^2 - 4ac = (-3)^2 - 4(1)(2)$
$a = 1$ $= 9 - 8 = 1$
$b = -3$
$c = 2$

$x^2 - 3x + 2 = (x-2)(x-1)$
Find 2 factors of 2 that add up to -3
$-2, -1$

Example 6: Factor $-12x^2 + 24x + 36$ $= -12(x^2 - 2x - 3) = \boxed{-12(x-3)(x+1)}$

$b^2 - 4ac = (-2)^2 - 4(1)(-3)$
$a = 1$ $= 4 - (-12)$
$b = -2$ $= 4 + 12$
$c = -3$ $= 16 = (4)^2$

$x^2 - 2x - 3 = (x-3)(x+1)$
Find 2 factors of -3 that add up to -2
$-3, 1$

Example 7: Factor $6x^2 + 13x - 15$

$b^2 - 4ac = (13)^2 - 4(6)(-15)$
$a = 6$ $= 169 + 360$
$b = 13$ $= 529 = (23)^2$
$c = -15$

-90
Find 2 factors of $(6)(-15)$ that add up to 13
$18, -5$

$6x^2 + 18x - 5x - 15$
$6x(x+3) - 5(x+3)$
$\boxed{(6x-5)(x+3)}$

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$16x^4 - 1$

$x^2 = y \qquad 16(x^2)^2 - 1$

$= 16y^2 - 1 = (4y)^2 - (1)^2 = (4y-1)(4y+1)$

$= (4x^2-1)(4x^2+1)$

$= ((2x)^2 - (1)^2)(4x^2+1)$

$= \boxed{(2x-1)(2x+1)(4x^2+1)}$

Example 8: Factor $16x^3 - 25x$ $= x(16x^2-25) = x((4x)^2-(5)^2)$

$a^2-b^2=(a-b)(a+b)=$ $\boxed{x(4x-5)(4x+5)}$

Example 9: Factor $30x^3 - 3x^2 - 9x$ $= 3x(10x^2-x-3) = \boxed{3x(2x+1)(5x-3)}$

ax^2+bx+c

$b^2-4ac=(-1)^2-4(10)(-3)$
$= 1+120 = 121 = (11)^2$

$a=10$
$b=-1$
$c=-3$

$10x^2-x-3$

Find 2 factors of $10(-3)=\underline{-30}$ that add up to $\underline{-1}$

$-6, 5$ ✓

$\underbrace{10x^2+5x}\underbrace{-6x-3}$

$5x(2x+1)-3(2x+1)$

$(2x+1)(5x-3)$

Example 10: Factor $x^4 + 3x^2 - 4$

$(x^2)^2+3x^2-4$

$y=x^2$ $y^2+3y-4=(y-1)(y+4)$

$a^2-b^2=(a-b)(a+b)$ $=(x^2-1)(x^2+4)$

$=\boxed{(x-1)(x+1)(x^2+4)}$

Example 11: Factor $-18x^3 + 54x^2 - 8x + 24$

$= -2(\underbrace{9x^3-27x^2}\underbrace{+4x-12})$

$= -2[9x^2(x-3)+4(x-3)]$

$= \boxed{-2(x-3)(9x^2+4)}$