

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$x^2 - 2x + 1 = (x-1)^2 \quad \begin{matrix} a=x \\ b=1 \end{matrix}$$

Math 1300

Section 4.4

Solving Equations by Factoring

Definition: The zero-product property says that if a and b are numbers and if $ab = 0$, then $a = 0$ or $b = 0$ (or both).

Definition: A quadratic equation is an equation that can be written $ax^2 + bx + c = 0$, where a , b , and c are numbers and $a \neq 0$.

Solving Quadratic Equations

To solve a quadratic equation, we must find all possible values for x that make

$$ax^2 + bx + c = 0$$

Factoring is usually a helpful way to solve quadratic equations.

To use factoring, move all nonzero terms to one side of the equal sign so that the other side is zero. Then use the zero-product property.

Examples:

1. Solve the equation: $x^2 - 5x - 24 = 0$

$$(x-8)(x+3) = 0$$

$$x-8=0$$

$$x+3=0$$

$$x=8$$

$$x=-3$$

2. Solve $2x^2 + 18x - 72 = 0$ for x .

$$2(x^2 + 9x - 36) = 0$$

$$2(x+12)(x-3) = 0$$

$2 \neq 0$

$$x+12=0$$

$$x-3=0$$

$$x=-12$$

$$x=3$$

3. $6x^2 - 27x = -12$

$$+12 \quad +12$$

$$6x^2 - 27x + 12 = 0$$

$$3(2x^2 - 9x + 4) = 0$$

$$3(x-4)(2x-1) = 0$$

$3 \neq 0$

$$x-4=0$$

$$2x-1=0$$

$$+1 \quad +1$$

$$\begin{array}{l} 2x^2 - 9x + 4 \\ \text{Find 2 factors of } 2(4)=8 \\ \text{that add up to } -9 \\ -8, -1 \\ \hline 2x^2 - 8x - x + 4 \\ \hline 2x(x-4) - 1(x-4) \end{array}$$

$$x = 4$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

$$(x-4)(2x-1)$$

$$4. \quad 16x^2 = 1$$

$$16x^2 - 1 = 0$$

$$(4x)^2 - (1)^2 = 0$$

$$(4x-1)(4x+1) = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$4x-1=0$$

$$\frac{4x}{4} = \frac{1}{4}$$

$$x = \frac{1}{4}$$

$$4x+1=0$$

$$\frac{4x}{4} = \frac{-1}{4}$$

$$x = -\frac{1}{4}$$

$$\frac{16x^2}{16} = \frac{1}{16}$$

$$\sqrt{x^2} = \sqrt{\frac{1}{16}}$$

$$x = \pm \frac{1}{4}$$

$$5. \quad -12x^2 = -17x + 6$$

$$+17x - 6 \quad +17x - 6$$

$$\ominus 12x^2 + 17x - 6 = 0$$

$$-1(12x^2 - 17x + 6) = 0$$

$$-1(3x-2)(4x-3) = 0$$

$$3x-2=0 \quad 4x-3=0$$

Find 2 factors of $(12)(6) = 72$
add up to -17

$-8, -9$

$$12x^2 - 8x - 9x + 6 = 0$$

$$4x(3x-2) - 3(3x-2)$$

$$(3x-2)(4x-3)$$

$$6. \quad x(x-2) = -1$$

$$x(x-2) + 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1=0$$

$$x = 1$$

$$7. \quad 9x^2 - 4 = 0$$

$$(3x)^2 - (2)^2 = 0$$

$$(3x-2)(3x+2) = 0$$

$$3x-2=0 \quad 3x+2=0$$

$$3x=2$$

$$x = \frac{2}{3}$$

$$3x=-2$$

$$x = -\frac{2}{3}$$

$$3x-2=0$$

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

$$4x-3=0$$

$$\frac{4x}{4} = \frac{3}{4}$$

$$x = \frac{3}{4}$$

$$9x^2 - 4 = 0$$

$$\frac{9x^2}{9} = \frac{4}{9}$$

$$\sqrt{x^2} = \sqrt{\frac{4}{9}}$$

$$x = \pm \frac{2}{3}$$

$$8. \quad x^2 + 16 = 0$$

No solution!

Solving Other Polynomial Equations

Solving other polynomial equations is done just like the quadratic equations: Set one side of the equation to zero, factor, use the zero-product property, and solve for x .

9. $4x^3 + 16x^2 + 15x = 0$

$$x(4x^2 + 16x + 15) = 0$$

$$x(2x+5)(2x+3) = 0$$

$$x = 0$$

$$2x+5=0$$

$$2x = -5$$

$$x = -5/2$$

$$2x+3=0$$

$$2x = -3$$

$$x = -3/2$$

$$4x^2 + 16x + 15$$

2 factors of $4(15) = 60$

that add up to 16 10, 6

$$4x^2 + 10x + 6x + 15$$

$$2x(2x+5) + 3(2x+5)$$

$$(2x+5)(2x+3)$$

10. $x^3 = 2x^2 + 99x$

$$-2x^2 - 99x - 2x^2 - 99x$$

$$x^3 - 2x^2 - 99x = 0$$

$$x(x^2 - 2x - 99) = 0$$

$$x(x-11)(x+9) = 0$$

$$x = 0$$

$$x = 11$$

$$x = -9$$

2 factors of -99

$-11, 9$

that add up to -2

11. $7x^3 - 14x^2 = 0$

$$7x^2(x-2) = 0$$

$$x^2 = 0$$

$$x-2=0$$

$$7 \neq 0$$

$$x = 0$$

$$x = 2$$

12. $x^3 + 18 = 2x^2 + 9x$

$$-2x^2 - 9x - 2x^2 - 9x$$

$$x^3 - 2x^2 - 9x + 18 = 0$$

$$x^2(x-2) - 9(x-2) = 0$$

$$(x-2)(x^2-9) = 0$$

$$(x-2)(x-3)(x+3) = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$x-2=0$$

$$x-3=0$$

$$x+3=0$$

$$x = 2$$

$$x = 3$$

$$x = -3$$

$$x^2 - 9 = 0$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

$$13. 30x^3 - 3x^2 - 9x = 0$$

$$3x(10x^2 - x - 3) = 0$$

$$3x(5x-3)(2x+1) = 0$$

$$3x = 0$$

$$x = 0$$

$$5x - 3 = 0$$

$$5x = 3$$

$$x = \frac{3}{5}$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$10x^2 - x - 3$$

$$\text{Product} = 10(-3) = -30$$

$$\text{Sum} = -1 \quad -6, 5$$

$$10x^2 - 6x + 5x - 3$$

$$2x(5x-3) + 1(5x-3)$$

$$(5x-3)(2x+1)$$

$$14. 2(x-1)^2 = 2x(x^2 - 20) + (8x^2 + 2)$$

$$2(x^2 - 2x + 1) = 2x^3 - 40x + 8x^2 + 2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$\begin{array}{r} 2x^2 - 4x + 2 \\ -2x^2 + 4x - 2 \\ \hline 0 \end{array} = \begin{array}{r} 2x^3 - 40x + 8x^2 + 2 \\ -2x^2 + 4x - 2 \\ \hline 0 \end{array}$$

$$0 = 2x^3 + 6x^2 - 36x$$

$$2x^3 + 6x^2 - 36x = 0$$

$$2x(x^2 + 3x - 18) = 0$$

$$2x(x+6)(x-3) = 0$$

$$x = 0$$

$$x + 6 = 0$$

$$x = -6$$

$$x - 3 = 0$$

$$x = 3$$

How to solve an equation:

- 1) Factor the expression.
- 2) Set each factor equal to 0.
- 3) Solve each simpler equation.

Solve each equation by factoring.

$$1. x^2 - 49 = 0$$

$$+49 \quad +49$$

$$\sqrt{x^2} = \sqrt{49}$$

$$x = \pm 7$$

$$x^2 - (7)^2 = 0$$

$$(x-7)(x+7) = 0$$

$$x-7=0$$

$$x+7=0$$

$$x=7$$

$$x=-7$$

$$2. \quad 9x^2 - 64 = 0$$

$$+64 +64$$

$$\frac{9x^2}{9} = \frac{64}{9}$$

$$\sqrt{x^2} = \sqrt{\frac{64}{9}}$$

$$x = \pm \frac{8}{3}$$

$$3. \quad x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0$$

$$x - 4 = 0$$

$$x = 4$$

$$4. \quad 2x^2 - 5x + 3 = 0$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(x-1)(2x-3) = 0$$

$$x-1 = 0$$

$$x = 1$$

$$2x - 3 = 0$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

$$\text{Product} = 2(3) = 6$$

$$\text{Sum} = -5 \quad -2, -3$$

$$5. \quad 3x^2 + 10x = 8$$

$$-8 \quad -8$$

$$3x^2 + 10x - 8 = 0$$

$$3x^2 + 12x - 2x - 8 = 0$$

$$3x(x+4) - 2(x+4) = 0$$

$$\text{Product} = 3(-8) = -24$$

$$\text{Sum} = 10 \quad 12, -2$$

$$(x+4)(3x-2)=0$$

$$x+4=0 \quad 3x-2=0$$

$$x = -4$$

$$3x=2$$

$$x = \frac{2}{3}$$

6. $-24x^2 - 72x - 48 = 0$

$$-24(x^2 + 3x + 2) = 0$$

$$-24(x+2)(x+1) = 0$$

$$x+2=0 \quad x+1=0$$

$$x = -2 \quad x = -1$$

$$\left\{ \begin{aligned} -24x^3 - 72x^2 - 48x &= 0 \\ -24x(x^2 + 3x + 2) &= 0 \end{aligned} \right.$$

$$-24x(x+2)(x+1) = 0$$

$$x=0 \quad x+2=0 \quad x+1=0$$

$$x = -2$$

$$x = -1$$

7. $x^3 - 2x^2 - 4x + 8 = 0$

$$x^2(x-2) - 4(x-2) = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$(x-2)(x^2-4) = 0$$

$$(x-2)(x-2)(x+2) = 0$$

$$(x-2)^2(x+2) = 0$$

$$x-2=0 \quad x+2=0$$

$$x = 2 \quad x = -2$$

8. $6x^2 + 13x - 15 = 0$

$$\text{Product} = 6(-15) = -90$$

$$\text{Sum} = 13 \quad -5, 18$$

$$(x+3)(6x-5) = 0$$

$$x+3=0 \quad 6x-5=0$$

$$x = -3$$

$$\frac{6x}{6} = \frac{5}{6}$$

$$x = \frac{5}{6}$$

$$\begin{aligned} &6x^2 + 18x - 5x - 15 \\ &= 6x(x+3) - 5(x+3) \\ &= (x+3)(6x-5) \end{aligned}$$

9. $16x^3 = 25x$

$$-25x \quad -25x$$

$$16x^3 - 25x = 0$$

$$x(16x^2 - 25) = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$x((4x)^2 - (5)^2) = 0$$

$$x(4x-5)(4x+5) = 0$$

$$x=0$$

$$4x-5=0$$

$$+5 \quad +5$$

$$4x+5=0$$

$$-5 \quad -5$$

$$\frac{4x}{4} = \frac{5}{4}$$

$$\frac{4x}{4} = \frac{-5}{4}$$

$$x = \frac{5}{4}$$

$$x = -\frac{5}{4}$$

10. $30x^3 - 3x^2 - 9x = 0$

$$3x(10x^2 - x - 3) = 0$$

$$3x(5x-3)(2x+1) = 0$$

$$x=0$$

$$5x-3=0 \quad 2x+1=0$$

$$5x=3 \quad 2x=-1$$

$$x = \frac{3}{5}$$

$$x = -\frac{1}{2}$$

Product = $10(-3) = -30$

Sum = $-1 \quad -6, 5$

$$10x^2 - 6x + 5x - 3 = 0$$

$$2x(5x-3) + 1(5x-3) = 0$$

$$(5x-3)(2x+1) = 0$$

11. $x^4 + 3x^2 - 4 = 0$

$$x^2 = y \quad y^2 + 3y - 4 = 0$$

$$x^4 = (x^2)^2 \quad (y-1)(y+4) = 0$$

$$= y^2 \quad (x^2-1)(x^2+4) = 0$$

$$(x-1)(x+1)(x^2+4) = 0$$

$$x-1=0 \quad x+1=0$$

$$x=1 \quad x=-1$$

$$x^2+4 \neq 0$$

$$-4 \quad -4$$

$$x^2 \neq -4$$

12. $4x^3 - 52x^2 - 3x + 39 = 0$

$$4x^2(x-13) - 3(x-13) = 0$$

$$(x-13)(4x^2-3) = 0$$

$$x-13=0 \quad 4x^2-3=0$$

$$+3 \quad +3$$

$$x=13$$

$$x = \frac{\sqrt{3}}{2}$$

$$x = -\frac{\sqrt{3}}{2}$$

$$\frac{4x^2}{4} = \frac{3}{4}$$

$$\sqrt{x^2} = \sqrt{\frac{3}{4}}$$

$$x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$