Math 1311
Final Test Review

Where:
Time:
What is covered? Chapters 1- 6
Number of questions: 20
Format: Multiple-choice
What you need to bring:

1. **Cougar Card** We do NOT accept Driver's license as a form of identification.
2. **Calculator** If you forget your calculator, you will have to take the test without. There are NO spares. Bring extra batteries too.
3. **Pencil** and eraser
4. **Popper** Same that you used for poppers during the semester.

1. The function $M(P, r, t) = \frac{Pr(1+r)^t}{(1+r)^t - 1}$ gives the monthly payment for a loan of P dollars at a monthly interest rate of r as a decimal if the loan is to be paid off in t months. Suppose you borrow \$5000 to buy a car and wish to pay off the loan over 3 years. Take the prevailing monthly rate to be 0.58%. Use function notation to show your monthly payment and then calculate its value.

MIR = 0.58% $r = 0.0058$
3years = 36 months

$M(5000, 0.0058, 36)$

$= \frac{5000 \times 0.0058 \times (1+0.0058)^{36}}{(1+0.0058)^{36} - 1} = \boxed{\$154.2}$

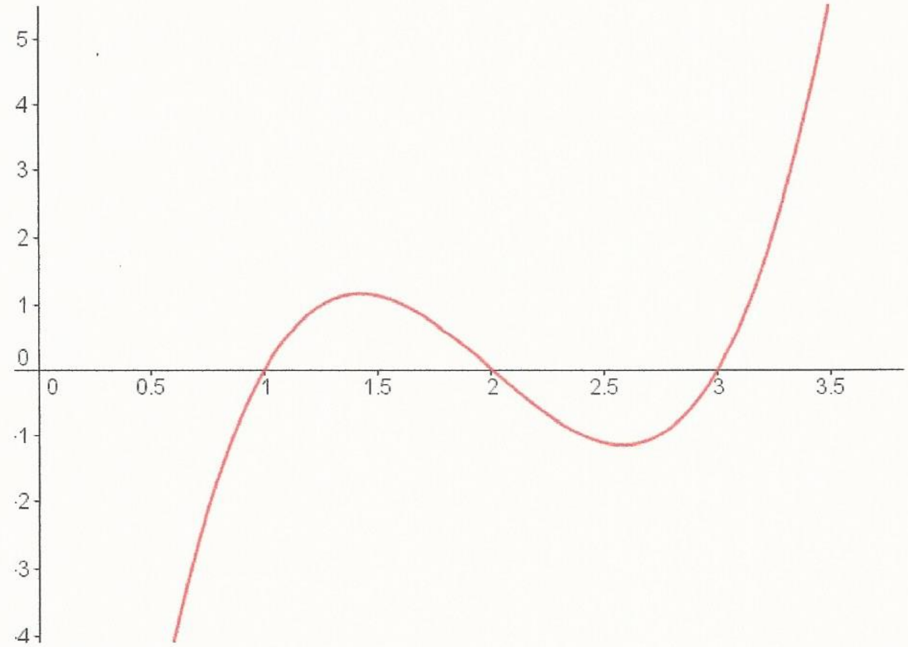
2. The following table gives values for a function $N = N(t)$. Calculate the average rate of change from $t = 20$ to $t = 30$. Use your answer to estimate the value of $N(27)$.

t	10	20	30	40	50	60	70
N = N(t)	17.6	23.8	44.6	51.3	53.2	53.7	53.9

$ARC = \frac{44.6 - 23.8}{30 - 20} = \boxed{2.08}$

$N(27) = 23.8 + 7(2.08) = \boxed{38.4}$

3. The following (on the next page) is a graph of function $f(x) = 3(x-1)(x-2)(x-3)$.



Where is the graph

a) decreasing and concave up? from 2 to 2.5

b) increasing and concave up? from 2.5 to ∞

4. You currently have \$500 in a piggy bank. You add \$37 to the bank each month. Find a formula that gives the balance B, in dollars, in the piggy bank after t months.

$B(t) = 500 + 37t$

5. It is a fact that the function ${(2 + 3^{-x})}/{(5 - 3^{-x})}$ has a limiting value. Use a table of values to estimate the limiting value.

The limiting value $= .4$

6. Suppose the function $f(x) = x^2 - 8x + 21$ describes a physical situation that only makes sense for the whole numbers between 0 and 20. For what value of x does f reach a minimum, and what is that minimum value.

$x = 4 \quad f(4) = 5$

7. Solve for k: $2k + m = 5k + n$.

$-2k \qquad -2k$

$m = 3k + n$

$-n \qquad\qquad -n$

$\frac{m-n}{3} = \frac{3k}{3}$

$\boxed{k = \frac{m-n}{3}}$

8. Use the crossing-graphs method to solve the given equation.

$$\underbrace{x+\sqrt{x+1}}_{Y_1}=\underbrace{\sqrt{x+2}}_{Y_2}$$

Window:
Xmin = 0
Xmax = 2
Ymin = -1
Ymax = 3

$\boxed{x = .37}$

9. The builder's Old Measurement was instituted by law in England in 1773 as the way to estimate the total tonnage T of a wooden ship from its beam width W and length L, both measured in feet. The formula is $T=\frac{(L-0.6W)W^2}{188}$. In this problem we consider wooden ships of length 150 feet.
   a) Make a graph of T versus W including beam widths up to 250 feet.
   b) What is the maximum tonnage for a ship of this length?
   c) What is the maximum tonnage of a ship whose width is no more than half its length?

(a) $T=\frac{(150-0.6W)W^2}{188}$

(b) 7387.71

(c) 3141.6

Window:
Xmin = 0   Ymin = 0
Xmax = 250   Ymax = 8000

10. A ladder leans against a wall so that its slope is 1.75. The top of the ladder is 9 vertical feet above the ground. What is the approximate horizontal distance from the base of the ladder to the wall? (Assume that the positive direction points from the base of the ladder toward the wall.)

$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$1.75 = \frac{9}{\text{run}}$

$\text{run} = \frac{9}{1.75} = \boxed{5.14 \text{ feet}}$

11. Find the slope of the line through the points $(\overset{x_1}{2},\overset{y_1}{2})$ and $(\overset{x_2}{4},\overset{y_2}{1})$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-2}{4-2}=\frac{-1}{2}=\boxed{-\frac{1}{2}}$$

12. If you take a brisk walk on a flat surface, you will burn about 258 calories per hour. You have just finished a hard workout that used 700 calories.

a) Find a formula that gives the total calories burned if you finish your workout with a walk of h hours.

b) How long do you need to walk at the end of your workout in order to burn a total of 1100 calories?

(a) $B = 700 + 258t$

(b) $\underbrace{700 + 258t}_{Y_1} = \underbrace{1100}_{Y_2}$

$t = 1.55$ hour or 93 minutes

13. The table below shows the number A, in millions, of motor vehicle accidents from year 2004 to year 2008.

$t =$ 0, 1, 2, 3, 4, 5

Date	2004	2005	2006	2007	2008
A=millions of accidents	10.9	10.7	10.4	10.6	10.2

2009

a) Find the equation of the regression line for A as a function of t, where t shows the number of years after 2004. linear

b) Express, using functional notation, the number of accidents in 2009, and then estimate that value.

(b) $A(5) = \boxed{10.11}$

(a) $Y = -.15x + 10.86$

or $A(t) = -.15t + 10.86$

14. A certain population grows by 23% per decade. What is its annual growth rate?

$a = 1 + 0.23 = 1.23$ per decade

$(1.23)^{1/10} = 1.021$ i.e. $\boxed{2.1\%}$

15. Suppose that f is an exponential function with decay factor 0.094 and that $f(0) = 400$.

a) Find a formula for f.

b) Find $f(2)$.

(a) $f(x) = 400 \times 0.094^x$

(b) $f(2) = 400 \times 0.094^2 = \boxed{3.53}$

16. A quantity increases by 5% for each of 10 years. What is the percentage increase over the 10-year period?

$1+0.05 = 1.05$

$(1.05)^{10} = 1.63$

$1.63 - 1 = .63 \rightarrow \boxed{63\%}$

17. The following table shows the income from sales of a certain magazine, measured in thousands of dollars, at the start of the given year.

t	0	1	2	3	4	5	6	7
Year	2005	2006	2007	2008	2009	2010	2011	2012
Income	7.76	8.82	9.88	10.94	12.00	13.08	14.26	15.54

Over an initial period the sales grew at a constant rate, and over the rest of the time the sales grew at a constant percentage rate. Calculate differences and ratios to determine what these time periods are, and find the growth rate or percentage growth rate, as appropriate.

$\frac{8.82-7.76}{1} = 1.06$

$\frac{9.88-8.82}{1} = 1.06$

$\frac{10.94-9.88}{1} = 1.06$

$\frac{12.00-10.94}{1} = 1.06$

$\frac{13.08-12.00}{1} = 1.08$

$\frac{14.26-13.08}{1} = 1.18$

$\frac{15.54-14.26}{1} = 1.28$

$\frac{13.08}{12} = 1.09$

$\frac{14.26}{13.08} = 1.09$

$\frac{15.54}{14.26} = 1.09$

Growth is linear from 2005 to 2009, exponential after that.

18. The table below gives the average number *N* of earthquakes of magnitude at least *M* that occur each year worldwide.

Magnitude M	6	6.1	6.6	7	7.3	8
Number N with magnitude at least M	95.8	77.8	27.6	12.1	6.5	1.5

a) Find an approximate exponential model for the data.

b) How many earthquakes per year of magnitude at least 5.5 can be expected?

(a) $N = 24671098.02 \times (.1254)^{M}$ (b) 271.05

$N = \frac{K}{1+be^{-rt}}$ $b = \frac{K}{N(0)} - 1$ $r = \ln a$

$r = .058$

$K = 1000$ $b = \frac{1000}{250} - 1 = 3$

$N(0) = 250$

$a = 1 + 0.06 = 1.06$

19. We begin selling a new magazine in a small town. Initial sales are 250 magazines per month. We believe that in the absence of limiting factors, our sales will increase by 6% per month, but the size of the town limits our total sales to 1000 magazines per month.

   a. Construct a logistic model for our magazine sales under these conditions.

   b. When can we expect sales to reach 750 magazines per month?

(a) $N = \frac{1000}{1+3e^{-0.058t}}$

(b) $\frac{1000}{1+3e^{-0.058t}} = 750$ $t = 37.88$ or 38 months

20. Let $f(x) = cx^4$. If x is doubled, by what factor is f increased?

$(2)^4 = \boxed{16}$

21. Model the following data with a power formula. You should be able to do this exercise quickly and easily without using your calculator.

x	1	2	3	4	5
y	1	8	27	64	125

$\boxed{y = x^3}$

22. Let $f(x) = x^2 - 1$ and $g(x) = 1 - x$. Find a formula for $f(g(x))$ in terms of x.

$f(g(x)) = (1-x)^2 - 1 = 1 - 2x + x^2 - 1 = \boxed{x^2 - 2x}$

23. Is $9.7x - 53.1x^4$ a polynomial? If it is a polynomial, give its degree.

Yes, 4

24. A rock is tossed upward and reaches its peak 2 second after the toss. Its location is determined by its distance up from the ground. What is the sign of velocity at each of the following times?

a) 1 second after the toss positive

b) 2 seconds after the toss 0

c) 3 seconds after the toss negative

25. What can be said about the graph of f if the graph of $\frac{df}{dx}$ is below the horizontal axis?

f is decreasing

26. The following table shows the population of reindeer on an island as of the given year.

t	0	5	10	15
Date	1945	1950	1955	1960
Population	40	165	678	2793

We let t be the number of years since 1945, so that $t = 0$ corresponds to 1945, and we let $N = N(t)$ denote the population size.

a) Approximate $\frac{dN}{dt}$ for 1955 using the average rate of change from 1955 to 1960.

b) Use your work from part a) to estimate the population in 1957.

(a) $\frac{dN}{dt} = \frac{2793 - 678}{15 - 10} = \boxed{423}$

(b) $678 + 2(423) = \boxed{1524 \text{ deer}}$

27. Suppose $f = f(x)$ satisfies $f(2) = 5$ and $f(2.005) = 5.012$. Estimate the value of $\frac{df}{dx}$ at $x = 2$.

$\frac{df}{dx} = \frac{5.012 - 5}{2.005 - 2} = \boxed{2.4}$

28. Solve the equation of change $\frac{df}{dx} = 5$ if the initial value of f is 3.

$f(x) = 5x + b$

$f(0) = 5(0) + b = 3 \Rightarrow b = 3$ $\boxed{f(x) = 5x + 3}$

29. Find an equilibrium solution of $\frac{df}{dx} = 2f - 6$.

$2f - 6 = 0$

$2f = 6$

$\boxed{f = 3}$

30. Find the common logarithm of $log10^{655.77}$ without using your calculator. Round your answer to two decimal places.

$\boxed{655.77}$