Math 1311
Section 1.1
Functions Given by Formulas

Topics:

- Using function notation
- Domain
- Answering questions when given a formula functions
- Using TI to compute function values and using the "Ans" feature of the calculator

What is a function?

Definition: A function is a rule that assigns to each element of one set (which we call the domain) exactly one element of some other set (which we call the range).

Example 1:

a. If you are driving across the country, you can write a function that gives the distance you have traveled. If you travel at 65 miles per hour, this could be your function:

$$Distance = rate\ of\ speed \times time = 65 \times time$$

D= 65t

b. If you work at a clothing store, you can write a function that gives the amount of money you earn in a week. If you make $8.50 per hour, this could be your function:

$$Pay = hourly\ rate \times number\ of\ hours\ worked = 8.50 \times number\ of\ hours\ worked$$

P= 8.5h

c. If you buy some clothing, you'll have to pay sales tax. You can write a function that gives the amount of sales tax. If sales tax is 8.25%, this could be your function:

$$Sales\ tax = cost\ of\ items \times tax\ rate = cost\ of\ items \times 0.0825$$

S=C(.0825)

- Functions involve both independent and dependent variables.
  We can choose values for the independent variable, so long as they make sense in the function. The value of a dependent variable depends on the value chosen for the independent variable.
- We assign letters for the independent and dependent variables. You can use any letter you like, subject to a couple of rules.

  1. You must precisely define your variables (including units)
  2. A letter can stand for only one quantity in your function.

With this in mind, we can rewrite the function above using mathematical notation:

a. $d = 65t$

b. P=8.50h

c. T=(0.0825)c

In $d = 65t$, the independent variable is t and the dependent variable is d.
Name the independent variable and the dependent variable in the other two functions.

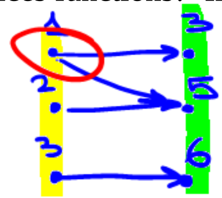
Example 2: Are the following correspondences functions? If not, explain why.

a) (1,3) (2,5) (1,5) (3,6)

b) (1,2) (3,4) (5,6) (7,8)

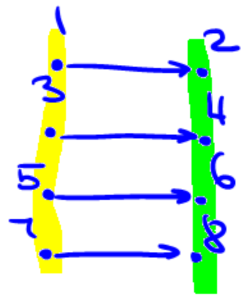
c) (1,2) (2,2) (3,4) (5,6)

a)



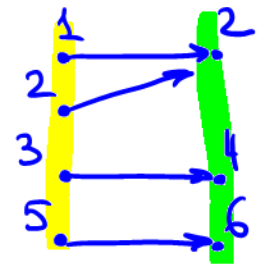
Not a function

b)



YES!

c)



YES!

Definition: The domain is the set of values that work in the function.

In each of our three cases of Example 1, only positive numbers make sense. You can't drive or work a negative number of hours and clothing can't have a negative cost.
We would write the domain as $[0, \infty)$ for each. This notation is called interval notation.

Here is a summary of interval notation:

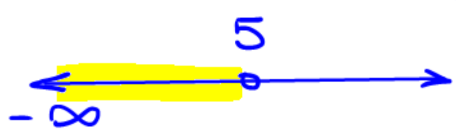
$(-3, 5)$ means all x such that $-3 < x < 5$

$[-3, 5]$ means all x such that $-3 \leq x \leq 5$

$[-3, 5)$ means all x such that $-3 \leq x < 5$

$[-3, \infty)$ means all x such that $x \geq 3$

$(-\infty, 5)$ means all x such that $x < 5$



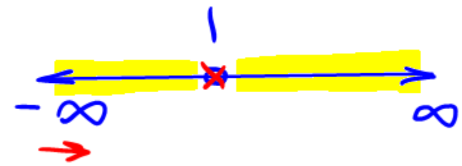
$(-\infty, \infty)$ means all real numbers

Some functions have specific restrictions on the domain:

Example 3:

a. State the domain $f(x) = \frac{5}{x-1}$

$x - 1 \neq 0$
$+1 \quad +1$
$x \neq 1$

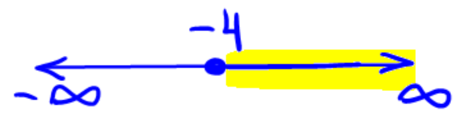


$(-\infty, 1) \cup (1, \infty)$

b. State the domain $f(x) = \sqrt{x + 4}$

$\sqrt{-4}$

$x + 4 \geq 0$
$-4 \quad -4$
$x \geq -4$



$[-4, \infty)$

We can evaluate a function at various values:

Example 4:

Suppose $f(x) = \frac{(x^4 - 2x + 3)}{4}$. Compute $f(0)$, $f(4)$, and $f(-3)$.

$-2^2 = -4$

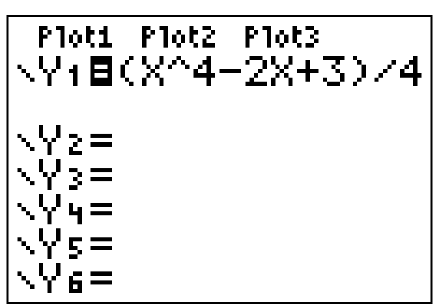
$(-2)^2 = 4$

$$f(0) = \frac{\cancel{(0)^4} - \cancel{2(0)} + 3}{4} = \frac{3}{4}$$

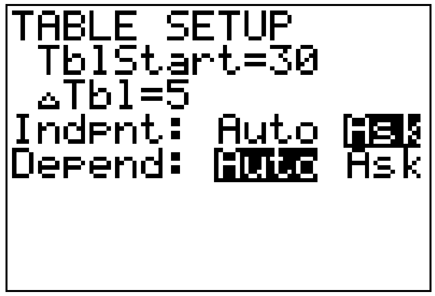
$$f(4) = \frac{(4)^4 - 2(4) + 3}{4} = \frac{256 - 8 + 3}{4} = \frac{251}{4} = 62.75$$

$$f(-3) = \frac{(-3)^4 - 2(-3) + 3}{4} = \frac{81 + 6 + 3}{4} = \frac{90}{4} = 22.5$$

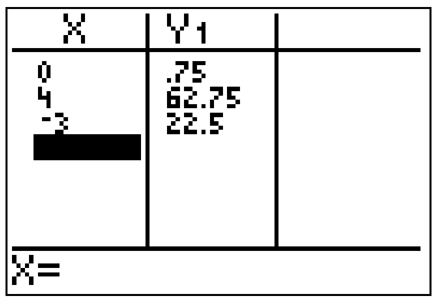
This can be done in the TI calculator by pressing  and entering the formula in Y_1



Then press   to set up a table. Make sure Indpnt is set to ask and Depend set to Auto. TblStart and ΔTbl do not matter here.



Finally, press   and then enter the desired input values.



Now for some **applications**:

Example 5: The time it takes David to travel from Houston to Denver is a function of the average speed travelled. The distance between the two cities is about 1200 miles. Suppose s is the average speed of David's car in miles per hour. Let $T = T(s)$ be the time it takes to get to Denver (in hours at the speed of s miles per hour). The formula for the function is

$$T(s) = \frac{1200}{s}$$

a. What does $T(60)$ represent?

b. Write (using function notation) an expression that shows the time it takes to get to Denver if David travels at an average speed of 75 miles per hour.

(a) Time it takes to get to Denver at speed of 60 mph.

(b) $T(75) = \frac{1200}{75} = 16$ hours

Example 6: Suppose your weekly pay is a function of the number of hours that you work per week. Let h represent the number of hours you work. Suppose your hourly pay rate is \$8.50. Then $P = P(h)$ is your weekly pay (before taxes!) in dollars. The formula for the function is

$$P(h) = 8.50h$$

a. What does $P(20)$ represent?

b. Write (using function notation) an expression that shows you pay for the week if you worked 34 hours.

(a) How much you get if you work 20 hours

(b) $P(34) = 8.50(34) = \$289$

Functions of Several Variables

Sometimes formulas involve more than one variable. In that case, you'll need to define each of the independent variables. This is called a function of several variables.

Example 7: You may be familiar with the formula for perimeter of a rectangle: $P = 2l + 2w$. This is a function of two variables, l and w. We can write it using function notation:

$$P(l, w) = 2l + 2w$$

$(5,6) \neq (6,5)$

The domain for this function is a set of ordered pairs, such as (10, 8).
Then $P(10,8)$ represents the perimeter of a rectangle with length 10 units and width 8 units.

Example 8: Suppose the amount of money, M, (given in dollars) spent at Lowe's for flooring is a function of the area of a room in square feet, x. The cost, c, of the flooring is given in dollars per square yard. Then $M = M(x, c)$ gives the cost of the flooring. The formula for this function is

$$M(x, c) = \frac{xc}{9}$$

a. What does $M(1000, 24)$ represent?

b. Write using function notation an expression for the cost of flooring for a room with 750 square feet with hardwoods that cost \$15.89 per square yard.

(a) The cost when you cover 1000 square feet at a cost of \$24 per sq. yard.

(b) $M(750, 15.89) = \frac{750(15.89)}{9}$

Example 9: If you borrow P dollars at a monthly interest rate r (written as a decimal) and wish to pay off the loan in t months, your monthly payment can be expressed as the function $M = M(p, r, t)$ given in dollars. The formula for this is

$$M(P, r, t) = \frac{Pr(1+r)^t}{(1+r)^t - 1}$$

Find the monthly payment if you borrow $15,000 at 4% interest for 4 years.

$P = 15{,}000$

$r = 4\% = .04$

$t = 4 \text{ years} = 48 \text{ months}$

$$M(15000, .04, 48) = \frac{(15000 * .04(1+.04)^{48}}{((1+.04)^{48} - 1)} = \$707.71$$

Using the "Ans" feature of the graphing calculator

Sometimes, evaluating functions of several variables using your calculator becomes complicated. If this is the case, you can evaluate part of your problem and then use that answer as you continue computing.

Locate the "Ans" button on your calculator. It's the 2nd of the key next to the enter button. To access the "Ans" feature, you'll need to press 2nd followed by the (-) key.

This calls up the previous answer displayed on your screen. So "Ans" is short for "Answer."

Redo Example 9 using "Ans" feature.

$$\frac{2^{5/8} - 7^{5/8}}{3^{5/8}}$$

Example 10: A ball is tossed upward from a building, and its upward velocity, V, in feet per second, is a function of the time t, in seconds, since the ball was thrown. The formula for this is $V(t) = 40 - 32t$. This function ignores air resistance. The function is positive when the ball is rising and negative when the ball is falling.

a. Find the velocity one second after the ball is thrown. Is the ball rising or falling then?
b. Find the velocity two seconds after the ball is thrown. Is the ball rising or falling then?
c. What is happening 1.25 seconds after the ball is thrown?
d. By how much does the velocity change from 1 second to 2 seconds? From 2 seconds to 3 seconds? From 3 seconds to 4 seconds? What does this mean?

(a) $V(1) = 40 - 32(1) = 8$ ft per second, rising

(b) $V(2) = 40 - 32(2) = -24$ ft per sec., falling

(c) $V(1.25) = 40 - 32(1.25) = 0$ the ball has reached its highest point

(d)

from 2 sec to 1 sec : $-24 - 8 = -32$ ft per second

from 3 sec. to 2 sec : $-56 - (-24) = -32$ ft per second

from 4 sec. to 3 sec : $-88 - (-56) = -32$ ft per second

Rate of change is constant!

Example 11: ~~Example 16:~~ The number N of deer present at time t (representing the number of years since the herd was introduced) on a certain wildlife refuge is given by the function

$$N(t) = \frac{12.36}{0.03 + 0.55^t}$$

- **a.** What does $N(0)$ represent? Calculate its value.
- **b.** How many deer would you expect to be on the reserve 5 years after the herd is introduced? Ten years? Fifteen years? Twenty years?
- **c.** How much increase in the deer population would you expect from the 10^{th} to the 15^{th} year?

(a) $N(0)$ = # of deer introduced to the refuge (initial # of deer)

(b) Use your calculator!

(c) $N(15) - N(10) = 410 - 380 = 30$ deer