**Math 1311**
**Section 1.2**
**Functions Given by Tables**

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Sometimes we work with functions for which we do not know a formula. In this case, many times we use a table of values to show the value of the function for specific values of the independent variables.

**Why tables?**
Advantages

- Tables are how we can organize experimental data for which the relationship is not known in formula form.
- Tables provide an easy to see relationship between quantities, since evaluating a formula is not always easy.
- It is often easier to spot trends in tabular data.

Disadvantages

- There are only finitely many values of the function known; we do not necessarily know what happens in the gaps.
- To use the table to guess what will happen in the future, you must assume that there is a pattern and that it will continue.

**Average Rate of Change** is a ratio of the change in function values to the change in values of the independent variable. Average rate of change of a function $f$ over the interval $(x_1, x_2)$ is defined to be

$$\boxed{AVERAGE\ RATE\ OF\ CHANGE = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

And AROC has units: the units of $f$ divided by the units of the $x$ variable.

This is a VERY IMPORTANT concept throughout this course.

**Limiting Values**

- Information about physical situations can sometimes show that limiting values are to be expected for functions that model those physical situations.
- The limiting value may be estimated from a trend established by the data.

There are 5 basic skills we need to learn from this section. They are

1. Reading a Table of Values
2. Averaging between two data points
3. Finding the Average Rate of Change for an Interval
4. Using the Average Rate of Change to Estimate the Values between data points
5. Recognizing a Limiting Value if it exists

We are going to practice these skills by looking at the tables for several physical situations

1945 1983 1985

**Example 1:** Women Employed Outside the Home

| **Year** | 1943 | 1946 | 1970 | 1980 | 1990 | 2000 |
|---|---|---|---|---|---|---|
| ***M*, # in millions** | 18.7 | 16.8 | 31.5 | 45.3 | 56.8 | 66.3 |

**a.** Explain what $M(1946)$ means and give its value.

$M(1946) = 16.8$ million of women.

**b.** Express the number of women employed outside the home in 1985 in function notation and find its value.

$$M(1985) = \frac{M(1980) + M(1990)}{2} = \frac{45.3 + 56.8}{2}$$

$= 51.05$ million of women.

**c.** Find the average rate of change for the interval 1980 ($x_1$) to 1990 ($x_2$).

$$AROC = \frac{M(1990) - M(1980)}{1990 - 1980} = \frac{56.8 - 45.3}{10}$$

$= 1.15$ million of women per year.

**d.** Use the average rate of change from part c. to estimate the number of women working outside the home in 1983.

$M(1983) = M(1980) + 3$ years of change

$= 45.3 + 3(1.15)$

$= 48.75$ million women

**e.** Find the average rate of change for this function from 1943 ($x_1$) to 1946 ($x_2$).

$$AROC = \frac{M(1946) - M(1943)}{1946 - 1943} = \frac{16.8 - 18.7}{3} = -.63$$

million of women per year

**f.** Use the average rate of change from part e. to estimate the number of women working outside the home in 1945.

$$M(1945) = M(1943) + 2 \text{ years of change}$$
$$= 18.7 + 2(-.63)$$
$$= 17.44 \text{ million of women}$$

**g.** The actual value $M(1945)$ is 19.0. Why is our average rate of change estimate so far off?

At the end of WW II many women stopped working.

**h.** Does the function appear to be increasing, decreasing or tending toward single value as the time goes on?

**Example 2:** The following table shows the number of deer in a wildlife preserve, $N$, as a function of the number of years since the herd was introduced.

| **Years, *t*** | ***N*** |
|---|---|
| 0 | 12 |
| 5 | 154 |
| 10 | 379 |
| 15 | 410 |
| 20 | 412 |
| 25 | 412 |

**a.** What does $N(15)$ mean and find its value?

$N(15) = 410$

15 years after the herd was intoduced there were 410 deer

**b.** Find the average rate of change of the deer population in the first five years on the reserve.

$$AROC = \frac{N(5) - N(0)}{5-0} = \frac{154-12}{5} = 28.4$$

$\approx 28$ deer per year

**c.** Use the AROC in part b. to estimate the deer population 2 years after the herd was introduced.

$N(2) = N(0) + 2$ years of change

$= 12 + 2(28) = 12 + 56 = 68$ deer

**d.** Make a chart that shows the average rate of change for each time interval in the table above.

| **Time interval** | **AROC, deer per year** |
|---|---|
| 0 to 5 | 28 |
| 5 to 10 | 45 |
| 10 to 15 | 6 |
| 15 to 20 | .4 |
| 20 to 25 | 0 |

$\leftarrow \frac{N(10)-N(5)}{5}$

**e.** Is there a limiting value for $N(t)$? What is it?

YES, 412 deer

**f.** What do you notice about the average rate of change as the value of $N$ gets close to the limiting value?

AROC becomes smaller & then $= 0$

**Example 3:** Gross Domestic Product

| **Year = t** | 1996 | 2000 | 2002 |
|---|---|---|---|
| **G = GDP, in millions of dollars** | 7.81 | 9.82 | 10.45 |

**a.** $G(1996)$

**b.** Estimate the gross domestic product in 1998 and express your answer in function notation.

**c.** Find the average rate of change of $G$ from 2000 to 2002.

**d.** Use the AROC from part c. to estimate the gross domestic product in 2010.