Math 1311
Section 1.3
Functions Given by Graphs

Representing a function with a graph gives a visual picture of how the function value depends on the value of the independent variable. This is the easiest way to spot changes and trends. However, drawing a graph is not a trivial exercise. If we have only a small number of data points, we have to guess what happens between them.

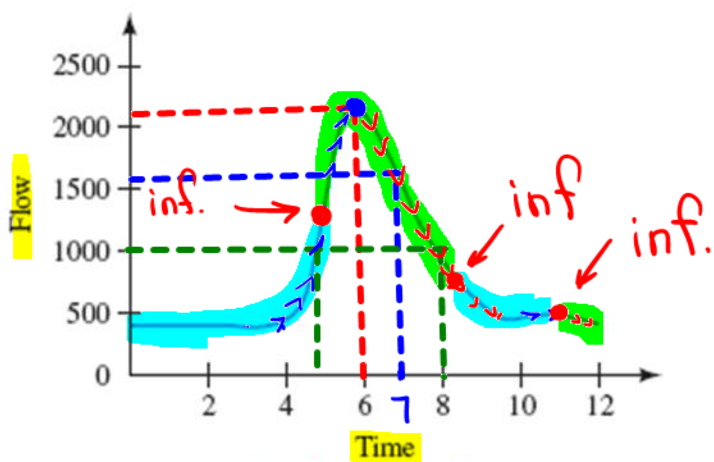
Skill #1 Reading a Graph
From a graph of y versus x (x on the horizontal axis, y on the vertical) you should be able to do find the following information:

- Find $f(x)$ for a particular value of x.
- Find the value (or values) of x for $f(x)$ a particular number.
- Explain how the function value changes with x, whether it increases, decreases, stays the same, or a combination of these.
- Calculate an average rate of change for the function between two x-values.

Finding values from a graph

- To evaluate a function given by a graph, locate the point of interest on the horizontal axis, move vertically to the graph, and then move horizontally to the vertical axis. The function value is the location on the horizontal axis.
- To find input value (or values) of x for a given $f(x)$, locate the point of interest on the vertical axis, read left and/or right to the curve (there may be multiple points), then read up/down to the horizontal axis to find the value.

Example 1: The graph below shows the mean flow F for the Arkansas River, in cubic feet of water per second, as a function of the time t, in months, since the start of the year. The flow is measured near the river's headwaters in the Rocky Mountains.



Flow for Arkansas River

7

a. Use functional notation to express the value of the flow at the end of July, and then estimate that value.

$F(7) = 1550$ cubic feet per second

ft^3/sec

b. When is the flow at its greatest?

At the end of June.

c. When is the flow 1000 cubic feet of water per second?

In May & in August.

Definition: A graph is increasing if the *y* values are getting bigger as you move from left to right. A graph is decreasing if the *y* values are getting smaller as you move from left to right.

d. Take a look at the graph. Where is a graph increasing and where is it decreasing? (Use interval notation)

Increasing from April to June & Oct. to Nov. $(4,6) \cup (10,11)$

Decreasing $(6,10) \cup (11,12)$

e. Estimate the average rate of change per month in the flow during the first two months of the year.

0

f. Estimate the average rate of change per month in the flow between April and June.

$F(4) = 480$

$F(6) = 2100$

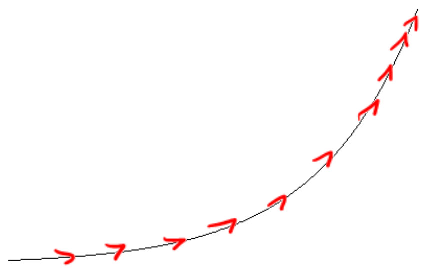
$$AROC = \frac{F(6) - F(4)}{6-4} = \frac{2100 - 480}{2} = 810$$

$ft^3/month$

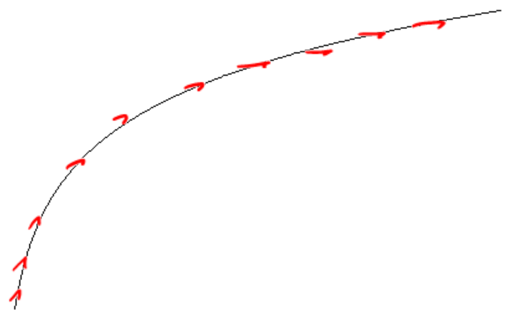
Skill #2 Recognizing and Interpreting Concavity on a graph.

If a graph is not just a straight line, pieces of the curve are either "bent upward" or "bent downward"
We call a curve shaped like a wire bent upward concave up.
A graph is called concave down if it is shaped like a wire bent downward.

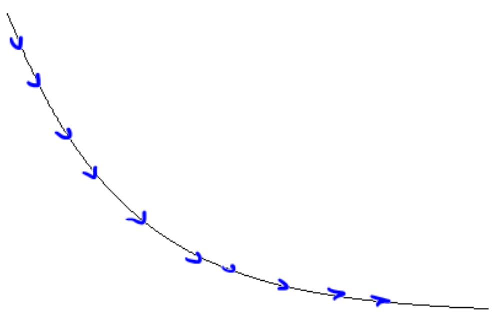
A function can be increasing or decreasing with either of the above concavities.



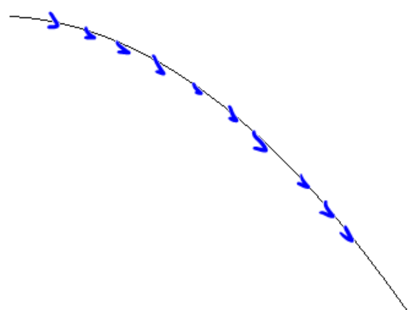
Concave Up and Increasing



Concave Down and Increasing



Concave Up and Decreasing

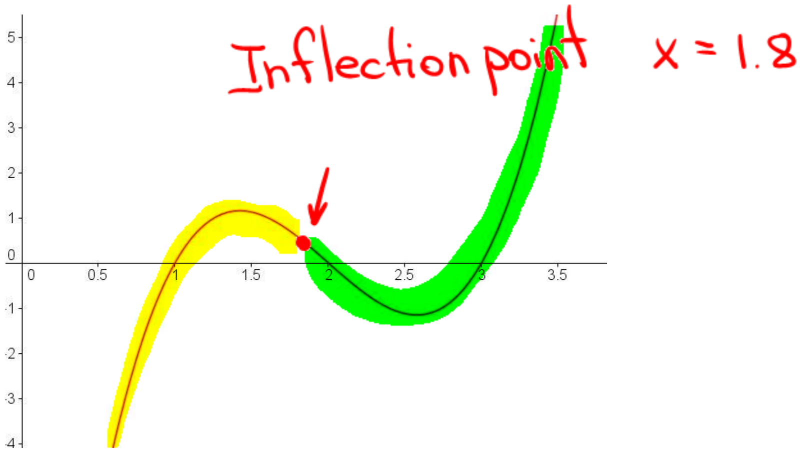


Concave Down and Decreasing

Concavity relates to how the average rate of change is changing! It seems like too many "changes", but the change in the AROC is important if your graph means something!

Definition: An inflection point is a point on the graph where the curve changes concavity.

Example 2:



Example 1(cont.): Referring to the graph from example 1 on the interval [0, 12], state where the graph is concave up and where it is concave down.

Concave up $(0,5) \cup (8,11)$

Concave down $(5,8) \cup (11,12)$

Inflection points: $t = 5, 8, 11$