Math 1311
Section 1.4
Functions Given by Words

Often times, the relationship between two quantities is given with a verbal description. We must then be able to use the verbal description to make sense out of various function values or even represent the function with a formula.

Skill #1 Creating a table of values for a function defined by words.

$t = 0$

Example 1: There are initially 1000 bacteria in a petri dish. The bacteria reproduce by cell division and the number of bacteria in the dish doubles in one hour. Make a chart of the number of bacteria as a function of time in hours.

Time (hours)	Number of Bacteria
0	1000
1	2000
2	4000
3	8000
4	16000

$3.5 \rightarrow$

$w = 0$

Example 2: The original price of a couch at The Home Store is $1200. The Home Store is going out of business and is getting rid of its inventory by having a sale. The price of furniture is reduced 10% each week of the sale. Make a chart of the cost of the couch as a function of the week of the sale.

Week Number	Price of Couch
0	1200
1	1080
2	972
3	874.80
4	787.32

$10\% \rightarrow .1$

$1200(1 - .1) = 1200(.9)$

$1080(1 - .1) = 1080(.9)$

$1200(.9)(.9)$

$972(.9)$

Skill #2 Comparing a function given by words to a given formula.

Example 1 Revisited: Let t = time in hours and $N = N(t)$ be the number of bacteria in the dish after t hours.

a. Verify that the formula $N(t) = 1000 * 2^t$ gives the correct values for the times in the table.

$N(0) = 1000 \cdot 2^0 = 1000$ ✓

$N(1) = 1000(2^1) = 2000$ ✓

$N(2) = 1000(2^2) = 4000$ ✓

$N(3) = 1000 \cdot 2^3 = 8000$ ✓

$N(4) = 1000 \cdot 2^4 = 16000$ ✓

b. Calculate the number of bacteria after 3.5 hours using the formula.

$N(t) = 1000 \cdot 2^t \qquad N(3.5) = 1000 \cdot 2^{3.5} = 11314$ bacteria

Skill #3 Translating a function given by words in a formula.

Basic Procedure for a Word Problem

1. Identify the function – what are the independent and dependent quantities?
2. Describe the function in a mathematical word sentence. Function value is equal to some combination of the independent quantities.
3. Assign variables, and write down what each variable stands for, including units.
4. Write the word sentence from part 2 in math terms with the given variables.
5. Solve the math problem; express your answer in the appropriate units.
6. Check to see if your answer makes sense!

Example 3: You pay $300 per ounce for silver and $21 per ounce for turquoise for making jewelry. Your completed pieces are sold for $500 per ounce.

a. Find the profit from the sale of a piece of jewelry made from 4 ounces of silver and 1 ounce of turquoise.

b. Find a simplified formula for $P(s,t)$ = the profit from the sale of a piece of jewelry made of s ounces of silver and t ounces of turquoise.

c. Evaluate and interpret $P(\overset{s}{3.8}, \overset{t}{1.4})$.

(a) Net profit = Revenue − Initial investment

= Profit per item × Number of items sold − Initial investment

$P = (4+1)(500) - 4(300) - 1(21) = \$1{,}279$

(b) s = # of ounces of silver used

t = # of ounces of tourquoise used

$P(s,t) = (s+t)\,500 - [300s + 21t]$

$= 500s + 500t - 300s - 21t = 200s + 479t$

$P(s,t) = \underline{200s + 479t}$

(c) $P(\overset{s}{3.8}, \overset{t}{1.4}) = 200(3.8) + 479(1.4)$ \$1,430.6

Example 4: The world record for a certain swimming event was 63.2 seconds in 1950, and each year thereafter the record has decreased by 0.4 seconds.

a. Give a formula for $R(x) =$ the record time x years after 1950.

b. Evaluate and interpret $R(4)$.

c. Evaluate and interpret $R(58)$.

d. Comment on the reality of using this model for large values of x.

x = # of years since 1950

1950 → $x = 0$
1951 → $x = 1$
etc.

(a) $R(x) = 63.2 - 0.4x$

(b) $R(4) = 63.2 - 0.4(4) = 61.6$ sec WR in 1954

(c) $R(58) = 63.2 - 0.4(58) = 40$ sec. WR in 2008

(d) in 158 years

$R(158) = 63.2 - 0.4(158) = 0$!

not possible

Example 5: A party cruise can be booked for a flat fee of \$1500 plus \$350 per person. However, discounts are given for large groups. For each person in excess of 10 that is part of the group, the price per person (for all people) decreases by \$15. Let $C(x)$ represent the total cost for a group of x people book this cruise.

a. Find and interpret $C(8)$.

b. Find and interpret $C(12)$.

c. Find and interpret $C(15)$.

d. Find a simplified formula for $C(x)$. Note that there will actually be 2 different formulas.

(a) $C(x) = \text{flat fee} + (\text{fee per person})(\text{number of people})$

$C(8) = 1500 + 350(8) = \$4300$ cost for 8 people

(b) $C(12) = 1500 + (350 - 15)12$

$= 1500 + 335(12) = \$5520$ cost for 12 people

(c) $C(15) = 1500 + 335(15) = \$6520$ cost for 15 people

(d) $$C(x) = \begin{cases} 1500 + 350x & 1 \le x \le 10 \\ 1500 + 335x & x > 10 \end{cases}$$

Example 6: You own a motel with 30 rooms and have a pricing structure that encourages rentals of rooms in groups. One room costs $85 per night, two for $83 per night (each), and, in general, the rate per room per night is found by taking $2 off the base price of $85 for each additional room rented. Let $R(x, n)$ be your revenue when a group rents x rooms for n nights.

a. Find and interpret $R(2,3)$.

b. Find and interpret $R(4,2)$.

c. Give a simplified formula for $R(x, n)$.

1 room → $85

2 rooms → $83

3 rooms → $81

(a) $R(2,3) = 2(83)(3) = \$498$ 2 rooms for 3 nights

# of rooms — price per room — # of nights

(b) $R(4,2) = 4(79)(2) = \$632$ 4 rooms for 2 nights

(c) $R(x,n) = x[85 - 2(x-1)]\, n$

$= x(85 - 2x + 2) \cdot n$

$R(x,n) = x(87 - 2x) \cdot n$

1 room → 0 discount

2 rooms → 2

3 rooms → 4

$2(x-1)$

Example 7: Water that is initially contaminated with a concentration of 9 milligrams of pollutant per liter of water is subjected to a cleaning process which reduces the pollutant concentration by 25% each hour. Let $C(t)$ denote the concentration, in milligrams per liter, of pollutant in the water t hours after the purification process begins.

$1 - .25 = .75$

a. Find and interpret $C(3)$.

b. Give a formula for $C(t)$.

c. Find the concentration of pollutant after 4hours 30 minutes of cleaning.

Reduced by 25% means 75% is left (.75)

(a) After 1 hour $9(.75)$

After 2 hours $9(.75)(.75) = 9(.75)^2$

After 3 hours $9(.75)^2(.75) = 9(.75)^3$

$C(3) = 9(.75)^3 = 3.80$ milligrams per litter

(b) $C(t) = 9(.75)^t$

(c) $C(4.5) = 9(.75)^{4.5} = 2.47$ milligrams per litter

9*.75^(4.5)