Math 1311
Section 2.3
Solving Linear Equations

A linear equation is one whose graph is a line.
With one variable, they look like $2x + 3 = -5x - 2$, just "x", no x^2 or other power of x.

Skill #1 Solving linear equations with algebra

Balanced Equation Rules

1. You can add or subtract the SAME number from both sides of the equation.
2. You can multiply or divide both sides of the equation by the SAME NONZERO number.

Objective is to get "x" by itself on one side of the equation.

Order of operations

A. Distribute, if needed.
   Example: $3(x - 1) = 3$ becomes $3x - 3 = 3$.
B. Add like terms on each side, if needed.
C. Add or subtract terms to each side so all terms with x are on one side and numbers on the other.
D. Divide by coefficient of the "x" term.

Example 1: Solve for x:

$3(x - 3) = 2x + 7$

$3x - 9 = 2x + 7$

$-2x \quad -2x$

$x - 9 = 7$

$+9 \quad +9$

$x = 16$

Check: $3(16 - 3) = 2(16) + 7$

$3(13) = 32 + 7$

$39 = 39$ ✓

$-x + 2(3 - x) = 50$

$-x + 6 - 2x = 50$

$-3x + 6 = 50$

$-6 \quad -6$

$\frac{-3x}{-3} = \frac{44}{-3}$

$x = -\frac{44}{3}$

Skill #2 Writing linear equations from word problems.

Example 2: A rental car company charges $20 for insurance and fees and then $18 per day for a rental car. Write the formula that expresses the cost C of renting a car as a function of the number of days n which the car is kept.

$C(n) = 20 + 18n$

Example 3: A small business is considering hiring a new sales representative to market its product in a nearby city. Two pay scales are being considered.

Pay Scale 1: Pay the sales rep a base yearly salary of $15,000 plus a commission of 8% of total yearly sales. .08

Pay Scale 2: Pay the sales rep a yearly salary of $20,000 and plus a commission of 5% of yearly sales. .05

a. For each scale above, give a function formula to express the total yearly earnings S as a function of the total yearly sales y.
b. What amount of total yearly sales would result in the same total yearly earnings for the sales rep no matter which of the two pay scales is used?

(a) Pay Scale 1 : $S_1(y) = 15000 + .08y$

Pay Scale 2 : $S_2(y) = 20000 + .05y$

(b) $S_1(y) = S_2(y)$

$15000 + .08y = 20000 + .05y$

$-.05y \qquad -.05y$

$15000 + .03y = 20000$

$-15000 \qquad -15000$

$\frac{.03y}{.03} = \frac{5000}{.03}$

$y = \$166,666.67$

Skill #3 Manipulating formulas which are linear in one or more variables.

Example 4: Temperature can be measured in several different measuring systems. The most common are measuring in degrees Celsius or degrees Fahrenheit. To find the temperature in degrees Celsius, C, given the temperature in degrees Fahrenheit, F, use the formula

$$C = \frac{5}{9}(F - 32)$$

Suppose we want to convert in the other direction, we are given temperatures in degrees Celsius and we what to find their equivalents in degrees Fahrenheit. We can solve the formula above for F and find a formula for F as a function of C.

$9C = \frac{9}{9}\frac{5}{9}(F-32)$

$\frac{9C}{5} = \frac{5}{5}(F-32)$

$\frac{9}{5}C = F - 32$

$+32 \qquad +32$

$\frac{9}{5}C + 32 = F$ $\qquad$ $F = \frac{9}{5}C + 32$

Example 3 Revisited: The new sales rep wants a feel for how much he has to sell to receive a specific salary. Write a formula for the year's sales as a function of the salary S the sales rep earns for pay scale 1.

$S = 15000 + .08y$

$-15000 \qquad -15000$

$\frac{S-15000}{.08} = \frac{.08y}{.08}$

wants to make 50,000

$y = \frac{S-15000}{.08}$

$\frac{S-15000}{.08} = y$

Example 5: Solve each of the following for the indicated variable.

$a = cb + db - c$; solve for c

$a - db = cb - c$

$\frac{a-db}{b-1} = \frac{c(b-1)}{b-1}$

$c = \frac{a-db}{b-1}$

$tx^2 = t + 1$; solve for t

$tx^2 - t = 1$

$\frac{t(x^2-1)}{x^2-1} = \frac{1}{x^2-1}$

$t = \frac{1}{x^2-1}$

Now, how can we use these functions, and how can the calculator help us?

Skill #4 Solving linear equations with the calculator.

Example 5: Solve $3.55x - 25 = 1.44x + 133$ by graphing on the calculator.

Y_1 Y_2

Step 1: Go to Y-menu 

. Enter $3.55x - 25$ as Y_1 . Enter $1.44x + 133$ as Y_2 .

Step 2: Graph the two equations. You may need to find a suitable graphing window so that you can see where the two lines intersect.

For this example, set the Window as [-10,100] with step 10 for x values and [-30, 350] with step 25 for y values.

Step 3: Press 

. Scroll down to Intersect and press enter. This should take you to the graph of the two lines.

Step 4: The cursor will be blinking on one of the lines, and you'll have a prompt that reads "First curve?". Press enter.

Step 5: The cursor will jump to the other line, and you'll have a prompt that reads "Second curve?". Press enter.

Step 6: You should now have a prompt that reads "Guess?". Press enter.

Step 7: The answer will be the x coordinate of the point that is given

74.88