Math 1311
Section 2.4
Solving Nonlinear Equations

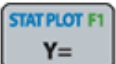$$x^3, x^5, x^{3/4} \qquad \sqrt{x}, \sqrt[3]{x} \qquad e^{2x}, 5^{3x+4}$$

Equations that involve powers (other than one), radicals (such as square roots), exponential terms, etc. are called nonlinear equations.

Some such equations have algebraic ways in which they can be solved. For most nonlinear equations, we will focus on graphical techniques (using the TI calculator).

Crossing Graphs Method

1. Press [STAT PLOT F1 / Y=], enter the one side of the equation in Y_1 and the other side in Y_2

2. Use the table to get a get an approximation of where the solution(s) is/are

3. Press [TBLSET F2 / WINDOW], set Xmin and Xmax so that the solution(s) are between them

4. Press [FORMAT F3 / ZOOM] [CATALOG / 0] to see the graph

5. Press [2ND] [CALC F4 / TRACE] and choose "5: intersect" and follow the on screen directions

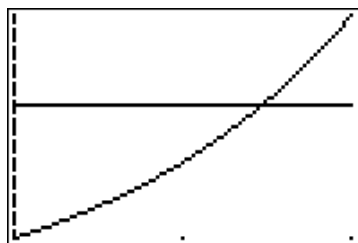**Example 1:** The screen shots below show solving the equation $x + 2^x = 50$. Y_1 Y_2

```
Plot1 Plot2 Plot3
\Y1 ◻X+2^X
\Y2 ◻50
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	Y2
0	1	50
2	6	50
4	20	50
6	70	50
8	264	50
10	1034	50
12	4108	50

Press + for ΔTbl

```
WINDOW
Xmin=4
Xmax=6
Xscl=1
Ymin=6
Ymax=50
Yscl=1
↓Xres=1
```

```
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
```

Intersection
X=5.4764953 Y=50

$$x = 5.48$$

Example 2: Solve each of the following using the crossing graphs.

a. $x^2 - x^3 + 3 = \frac{x^5}{20}$

$x =$ **1.74** Xmin = **0** Xmax = **2**

b. $x + 3^x = 20$

$x =$ **2.6** Xmin = **2** Xmax = **4**

c. $\frac{1+2^x}{1+3^x} = x^2$ This equation has 2 solutions, one positive and one negative.

$x =$ **-1.06** Xmin = **-2** Xmax = **0**

$x =$ **.88** Xmin = **0** Xmax = **1**

d. $e^{4-x} = x - 4$

$x =$ **4.57** Xmin = **4** Xmax = **5**

e. $20(1 + 2^x) = x$

$x =$ _____ Xmin = _____ Xmax = _____

no solution

Example 3: The temperature C of a fresh cup of coffee t minutes after it is poured is given by $C(t) = 125e^{-0.03t} + 75$. The coffee is cool enough to drink when its temperature is 150 degrees. When will the coffee be cool enough to drink?

$$125e^{-0.03t} + 75 = 150$$

Y_1 Y_2

$\approx 17 \, min.$

Example 4: If you borrow $5000 at an APR of r (as a decimal) from a bank that compounds interest continuously, and you plan to make monthly payments for 3 years to pay back the loan, then the monthly payment is $M(r) = \frac{5000(e^{(r/12)}-1)}{1-e^{-3r}}$. If your budget will allow for a $150 a month payment, what APR must you receive?

$$150 = \frac{5000\left(e^{r/12}-1\right)}{1-e^{-3r}}$$

Y_1 Y_2

$$r = .05 \qquad APR = 5\%$$