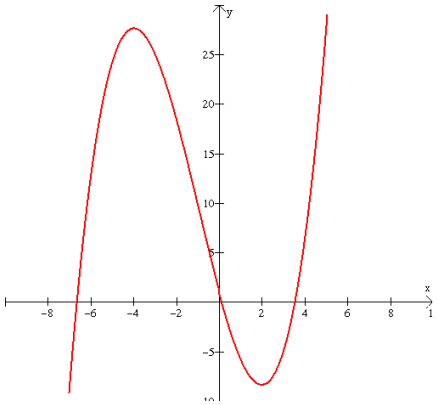
Math 1311
Section 2.6
Optimization

Optimization refers to finding the "best value" for a particular quantity, usually finding the largest or smallest function value for a particular situation.
A function $f(x)$ has a maximum (local maximum) at $x = c$ if $f(c)$ is bigger than $f(x)$ for any x in an interval around c.
A function $f(x)$ has a minimum (local minimum) at $x = b$ if $f(b)$ is smaller than $f(x)$ for any x in an interval around b.
The easiest way to find a local maximum (high point) or local minimum (low point) is to look at a graph of the function in the interval of interest.

Finding the maximum and/or minimum value of a function is called optimization.

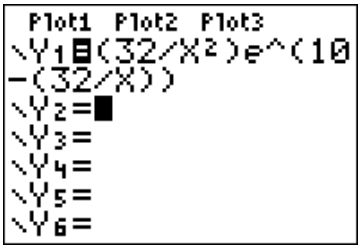


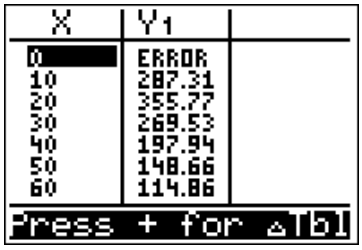
When we graph a function on our graphing calculator, the calculator can do the math necessary to find the maximum and minimum in the interval of interest.

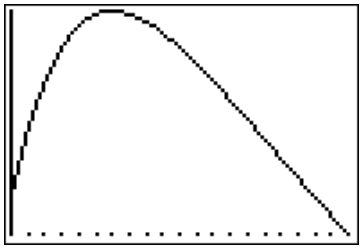
Skill #1 Finding a Max or Min in the graphing window

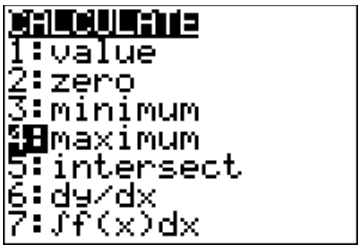
1. Put the function's formula in Y_1.
2. Use a table (we learned how to make these in the calculator earlier) to find an x range.
3. We are looking for where the y values change from increasing to decreasing or from decreasing to increasing.
4. Press TBLSET F2 WINDOW and set Xmin and Xmax based on step 2, then press FORMAT F3 ZOOM CATALOG 0
5. Press 2ND CALC F4 TRACE and choose "3: minimum" or "4: maximum"
   Then follow the on screen directions.

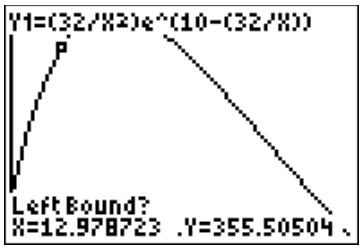
Example 1: Below we find the maximum of $f(x) = \left(\frac{32}{x^2}\right) e^{10-32/x}$.

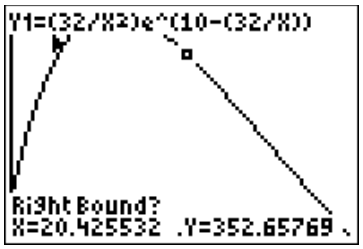


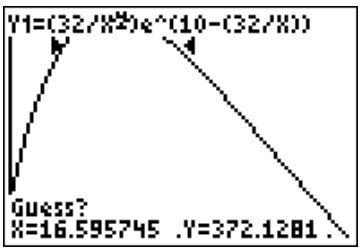


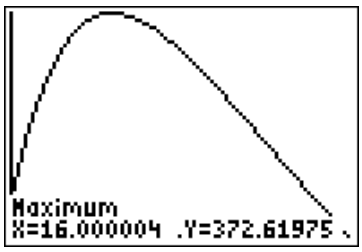












So the maximum point is about (16, 372.62).

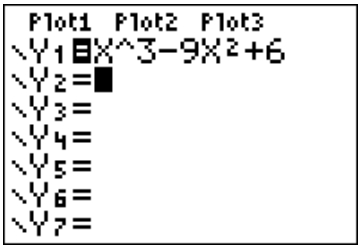
Example 2: A cannonball is fired into the air. The height of the cannonball (in feet) when the cannonball has traveled x feet horizontally is $(x) = x - 32\left(\frac{x}{250}\right)^2$. Find the maximum height of the cannonball. How far does the cannonball travel horizontally before achieving this height?

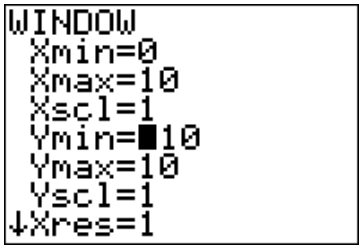
Skill #2 When a max or min occurs at and endpoint.

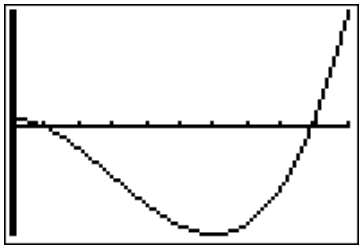
Some functions (for our purposes almost all functions) are guaranteed to have both a maximum and minimum value when looked at only on a closed, bounded interval $[a, b]$.

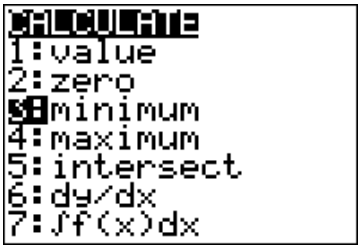
1. Put the function in Y_1
2. Press TBLSET F2 / WINDOW and set Xmin and Xmax as given in the problem
3. Press FORMAT F3 / ZOOM CATALOG / 0
4. If the max and/or min value occur at a peak or valley, use the technique discuss above to find this optimal value. If the max and/or min occur at the beginning or end of the graph, then while tracing the graph; enter the x – value of the appropriate endpoint.

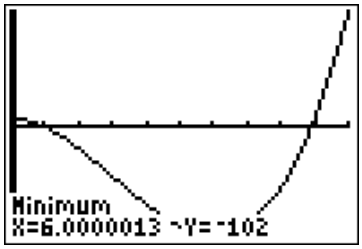
Example 3: Below we find the maximum and minimum of $y = x^3 - 9x^2 + 6$ on the interval $[0, 10]$.

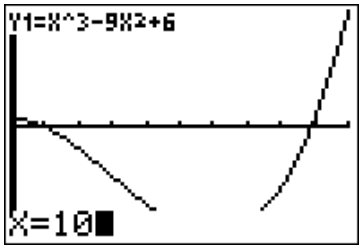


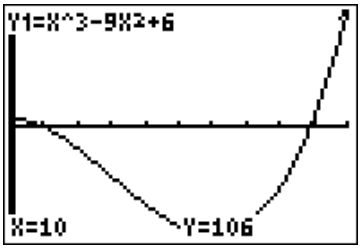












Example 4: Radium-223 is a radioactive substance that is itself a product of the radioactive decay of thorium-227. For one experiment, the amount A of radium present (measured in grams) after x days is given by $= 3(e^{-0.038x} - e^{-0.059x})$.

a. What was the largest amount of radium-223 present over the first 10 days of the experiment?
b. What was the largest amount of radium-223 present over the first 60 days of the experiment?
c. What was the smallest amount of radium-223 present over the first 60 days of the experiment?

Example 5: The manager of an employee health plan for a firm has studied the balance B, in millions of dollars, in the plan account as a function of x, the number of years since the plan was instituted. He has determined the account balance is very well modeled by the formula $B = 60 + 7x - 50e^{0.1x}$.

a. During the first 7 years of the plan, at what time was the balance greatest? What was that balance?
b. During the first 7 years of the plan, at what time was the balance lowest? What was that balance?