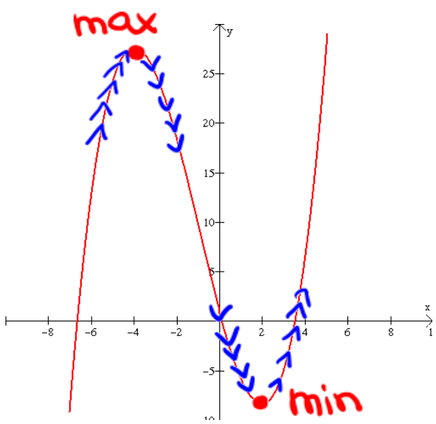
Math 1311
Section 2.6
Optimization

Optimization refers to finding the "best value" for a particular quantity, usually finding the largest or smallest function value for a particular situation.
A function $f(x)$ has a maximum (local maximum) at $x = c$ if $f(c)$ is bigger than $f(x)$ for any x in an interval around c.
A function $f(x)$ has a minimum (local minimum) at $x = b$ if $f(b)$ is smaller than $f(x)$ for any x in an interval around b.
The easiest way to find a local maximum (high point) or local minimum (low point) is to look at a graph of the function in the interval of interest.

Finding the maximum and/or minimum value of a function is called optimization.



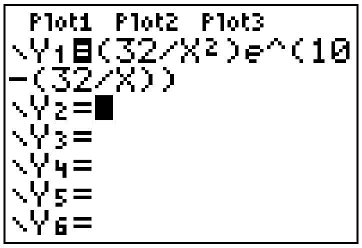
When we graph a function on our graphing calculator, the calculator can do the math necessary to find the maximum and minimum in the interval of interest.

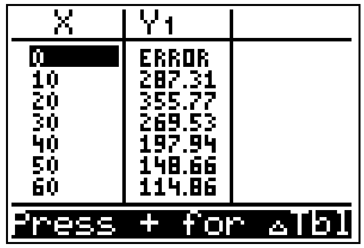
Skill #1 Finding a Max or Min in the graphing window

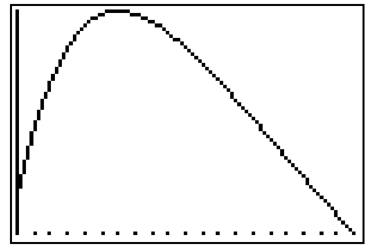
1. Put the function's formula in Y_1.
2. Use a table (we learned how to make these in the calculator earlier) to find an x range.
3. We are looking for where the y values change from increasing to decreasing or from decreasing to increasing.
4. Press [TBLSET F2 WINDOW] and set Xmin and Xmax based on step 2, then press [FORMAT F3 ZOOM] [CATALOG 0]
5. Press [2ND] [CALC F4 TRACE] and choose "3: minimum" or "4: maximum"

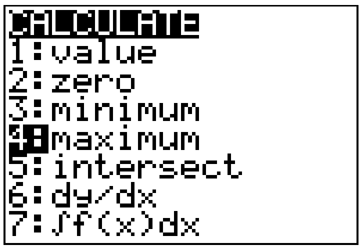
   Then follow the on screen directions.

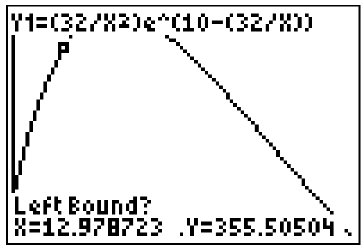
Example 1: Below we find the maximum of $f(x) = \left(\frac{32}{x^2}\right) e^{10-32/x}$.

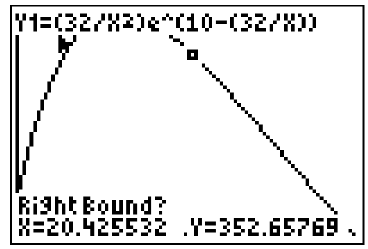


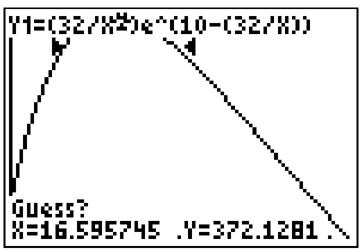


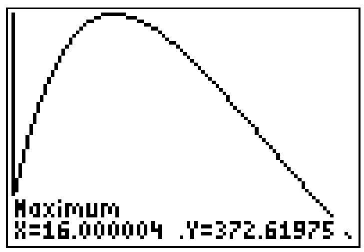






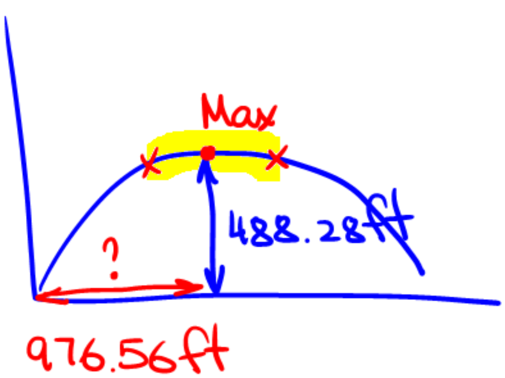






So the maximum point is about (16, 372.62). Max of f(x) = 372.62 at x = 16

Example 2: A cannonball is fired into the air. The height of the cannonball (in feet) when the cannonball has traveled x feet horizontally is $h(x) = x - 32\left(\frac{x}{250}\right)^2$. Find the maximum height of the cannonball. How far does the cannonball travel horizontally before achieving this height?

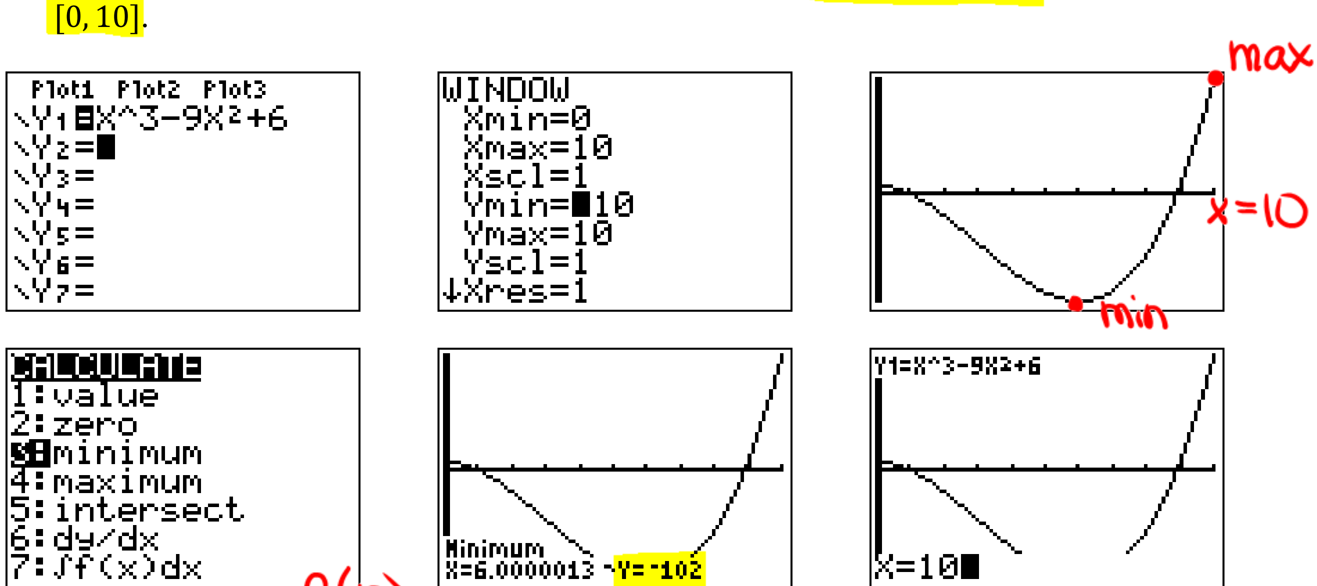


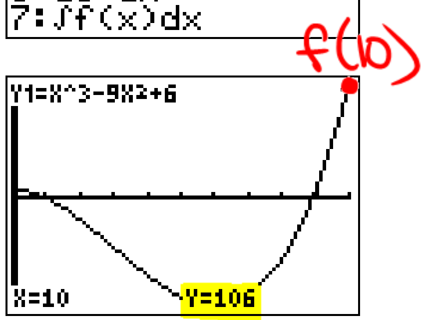
Skill #2 When a max or min occurs at and endpoint.

Some functions (for our purposes almost all functions) are guaranteed to have both a maximum and minimum value when looked at only on a closed, bounded interval $[a, b]$.

1. Put the function in Y_1
2. Press  and set Xmin and Xmax as given in the problem
   a b
3. Press  
4. If the max and/or min value occur at a peak or valley, use the technique discuss above to find this optimal value. If the max and/or min occur at the beginning or end of the graph, then while tracing the graph; enter the x – value of the appropriate endpoint.

Example 3: Below we find the maximum and minimum of $y = x^3 - 9x^2 + 6$ on the interval $[0, 10]$.





Min = -102

Max = 106

Example 4: Radium-223 is a radioactive substance that is itself a product of the radioactive decay of thorium-227. For one experiment, the amount A of radium present (measured in grams) after x days is given by $A = 3(e^{-0.038x} - e^{-0.059x})$.

$X_{min} = 0 \quad X_{max} = 10$

a. What was the largest amount of radium-223 present over the first 10 days of the experiment?

$X_{min} = 0 \quad X_{max} = 60$

b. What was the largest amount of radium-223 present over the first 60 days of the experiment?

c. What was the smallest amount of radium-223 present over the first 60 days of the experiment?

Y value

X value

(a) $A_{max} = .39$ grams at day 10

(b) $A_{max} = .48$ grams at day 21

(c) $A_{min} = 0$ grams at day 0 (at the beginning of experiment)

Example 5: The manager of an employee health plan for a firm has studied the balance B, in millions of dollars, in the plan account as a function of x, the number of years since the plan was instituted. He has determined the account balance is very well modeled by the formula $B = 60 + 7x - 50e^{0.1x}$.

$X_{min} = 0 \quad X_{max} = 7$

a. During the first 7 years of the plan, at what time was the balance greatest? What was that balance?

b. During the first 7 years of the plan, at what time was the balance lowest? What was that balance?

(a) After 3.36 years \$13.55 million

(b) After 7 years \$8.31 million