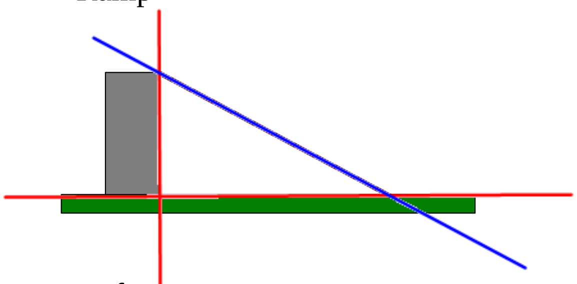
Math 1311
Section 3.1
The Geometry of Lines

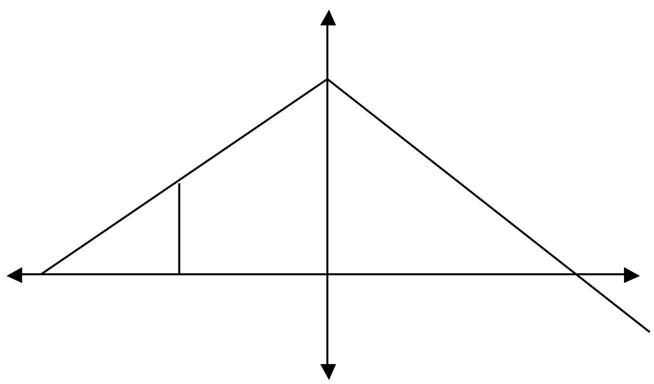
Approach One: A line is determined by two points

We can use coordinate axes to represent lines in the real world so we can use math to solve problems.

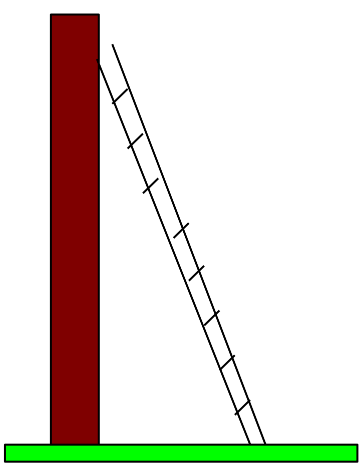
Ramp

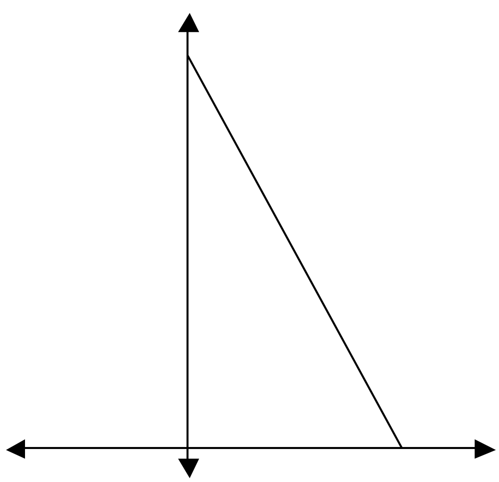


Roof Line



Ladder





Slope of a Line: The slope is defined to be the average rate of change for the function.

(x_1, y_1) (x_2, y_2)

$f(x_1)$ $f(x_2)$

$$slope = \frac{rise}{run} = \frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

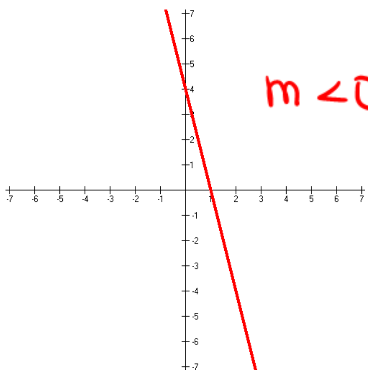
- For a given line, for whatever two points are selected, the average rate of change between then will be the same.
- Positive when the line rises from left to right.
- Negative when the line falls from left to right.
- Zero when the line is horizontal (no change in y)

- Slope can be used to calculate the change by the formula

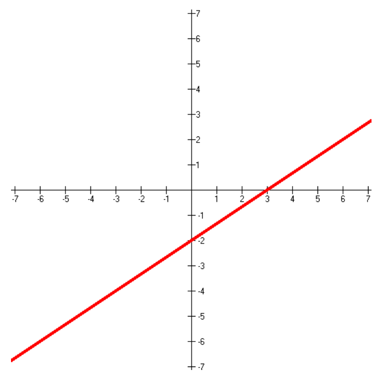
$$vertical\ change = m * horizontal\ change$$

rise = m · run

Example 1:



$m < 0$



$m > 0$

Example 2: Find the slope between the pair of points given:

a. $(3, 8)$ & $(-2, 14)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{Y}{X}$$

$$m = \frac{14 - 8}{-2 - 3} = \frac{6}{-5} = -\frac{6}{5}$$

$$\boxed{= -1.2}$$

b. $(3, -8)$ & $(-1, -22)$

$$m = \frac{-22 - (-8)}{-1 - 3} = \frac{-22 + 8}{-4}$$

$$= \frac{-14}{-4}$$

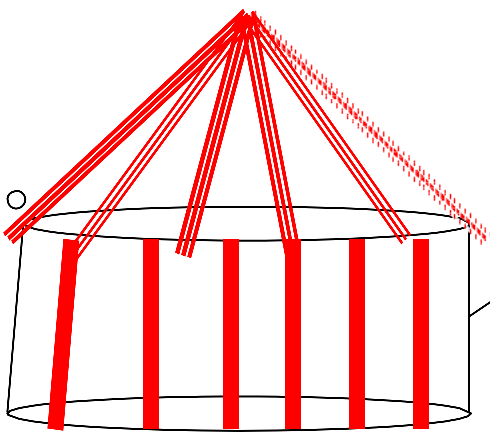
$$= \frac{14}{4} = \frac{7}{2}$$

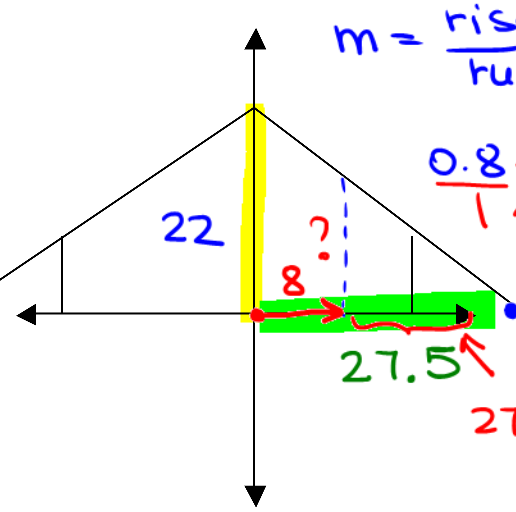
$$\boxed{= 3.5}$$

Approach Two – Using slope to define a line.

Example 3: The Circus Tent Problem.

At the center of the circus tent, the height is 22 feet. The slope of the tent roofline is 0.8.





$m = \frac{rise}{run}$

$\frac{0.8}{1} = \frac{22}{run}$

$run = \frac{22}{0.8}$

$= 27.5$

$27.5 - 8 = 19.5$

a. If you walk 8 feet from the center of the tent, how high is the roof?

$0.8 = \frac{rise}{run}$ $\quad \frac{0.8}{1} = \frac{rise}{19.5} \Rightarrow rise = 19.5(0.8) = 15.6\text{ft}$

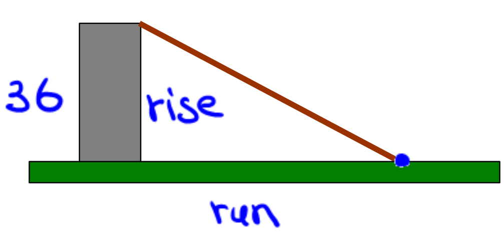
b. The tent is staked with ropes that follow the roofline. How far from the center of the tent are the stakes?

27.5

Example 4: The Ramp Problem.

The standard for a wheelchair ramp is one inch of vertical change for each 12 inches of horizontal change. If you need to build a wheelchair ramp to get from the ground to a porch which is 3 feet high, how far away from the porch does the ramp need to start?

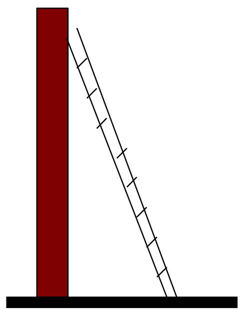
$m = \frac{rise}{run} = \frac{1}{12}$

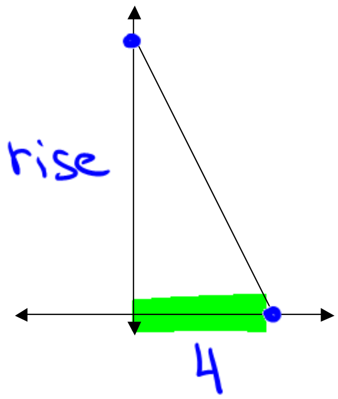


$\frac{1}{12} = \frac{36}{run}$ $\quad run = 12(36)$

$= 432\text{inches}$

Example 5: The Ladder Problem.
The base of a ladder is 4 horizontal feet from the wall. The slope of the line made by the ladder is –1.25. Find the vertical height of the top of the ladder.





$$m = -1.25$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$-1.25 = \frac{\text{rise}}{4}$$

$$\text{rise} = -1.25(4)$$

$$= -5$$

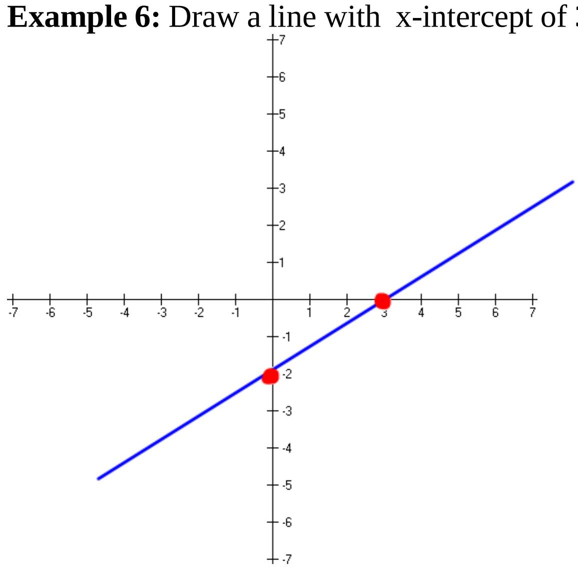
$$\boxed{\text{height} = 5\text{ft}}$$

Approach Three: Lines from Intercepts and Slope

Definition: The horizontal or x-intercept is the x value where a line crosses the x axis. The vertical or y-intercept is the y value where a line crosses the y-axis.

We can draw a line from both intercepts or from one intercept and a slope.

Example 6: Draw a line with x-intercept of 3 and y-intercept of – 2.



Example 7: Find the point where the line through $\overset{x_1}{1.2}, \overset{y_1}{3.1}$ $(1.2, 3.1)$ with slope -0.8 crosses the horizontal axis.

$(\overset{x_2}{x}, \overset{y_2}{0})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-0.8 = \frac{0 - 3.1}{x - 1.2}$$

$$\underbrace{-0.8}_{Y_1} = \frac{-3.1}{\underbrace{x - 1.2}_{Y_2}}$$

$$x = 5.075$$

$$\boxed{(5.075, 0)}$$

window: $[4, 6] \times [-2 \times 1]$

Example 8: Find the point where the line through $(-4, 3)$ with slope 3 crosses the vertical axis.

$(\overset{x_1}{-4}, \overset{y_1}{3})$

$(\overset{x_2}{0}, \overset{y_2}{y})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{y - 3}{0 - (-4)}$$

$$4 \cdot 3 = \frac{y - 3}{\cancel{4}} \cdot \cancel{4}$$

$$12 = y - 3$$

$$+3 \qquad +3$$

$$y = 15$$

$$\boxed{(0, 15)}$$

Example 9: Find the point with horizontal coordinate 6.6 that lies on the line through $(1, 5)$ with slope -1.1.

$(\overset{x_1}{1}, \overset{y_1}{5})$

$(\overset{x_2}{6.6}, \overset{y_2}{y})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\underbrace{-1.1}_{Y_1} = \frac{y - 5}{\underbrace{6.6 - 1}_{Y_2}}$$

$$5.6 \cdot -1.1 = \frac{y - 5}{\cancel{5.6}} \cdot \cancel{5.6}$$

$$-6.16 = y - 5$$

$$+5 \qquad +5$$

$$-1.16 = y$$

$$\boxed{(6.6, -1.16)}$$