Math 1311
Section 3.2
Linear Functions

A **linear function** is a function which has a constant rate of change, i.e. slope.
The slope is the amount of change in the function value when the independent variable increases by 1.
Suppose $y = f(x)$ is a function of x. Then:

$$\boxed{slope\ m = \frac{change\ in\ y}{change\ in\ x} = \frac{change\ in\ function}{change\ in\ variable}}$$

Equations
Slope – Intercept Form

- A linear function has formula $\boxed{y = f(x) = mx + b}$.
- m is the slope of the line.
- The point $(0, b)$ is the vertical (y) intercept.
- In practical terms, b represents the initial value of the output.

Point – Slope Form

- Suppose we know that a linear function has slope m and passes through the point (x_1, y_1), then the equation of the line can be written as $\boxed{y - y_1 = m(x - x_1)}$.
- From this equation, solving for y gives the equation of the linear function.

Example 1: Give the formula for the linear function described:

a. slope of 7 and y–intercept $(0, -2)$.

$m = 7$
$b = -2$

$y = mx + b$
$y = 7x + (-2)$
$\boxed{y = 7x - 2}$
$f(x) = 7x - 2$

b. slope of -4 and passes through the point (2, -3).

$m = -4$
$\overset{x_1}{(2}, \overset{y_1}{-3)}$

$y - y_1 = m(x - x_1)$
$y - (-3) = -4(x - 2)$
$y + 3 = -4x + 8$
$\;\;\,-3 \qquad\quad -3$
$\longrightarrow \boxed{y = -4x + 5}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

c. passes through the points $(0,4)$ and $(2,-6)$.

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{-6-4}{2-0} = \frac{-10}{2} = -5$

$y - y_1 = m(x - x_1)$

$y - 4 = -5(x - 0)$

$(0,4)$ - y-intercept $\quad b = 4$

$y - 4 = -5x$
$+4 \qquad +4$

$y = mx + b$

$\boxed{y = -5x + 4} \longleftrightarrow y = -5x + 4$

d. passes through the points $(-3,5)$ and $(7,24)$.

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{24-5}{7-(-3)} = \frac{19}{10} = 1.9$

$y - y_1 = m(x - x_1)$

$y - 5 = 1.9(x - (-3))$

$y - 5 = 1.9(x + 3)$

$y - 5 = 1.9x + 5.7 \rightarrow \boxed{y = 1.9x + 10.7}$
$+5 \qquad\qquad +5$

Example 2: Suppose that at the beginning of an experiment there are 500 bacteria present and that this number is decreasing at a rate of 75 bacteria per hour.

a. How can we tell that this relationship is linear?
b. Give a formula for N, the number of bacteria after h hours.

a) Linear functions have const. ROC

b) $y = mx + b \qquad m = -75$

$(0,500) \quad b = 500$

$N(h) = -75h + 500$

Example 3: A certain company manufactures widgets. Suppose that the cost of leasing the building, buying the equipment, but producing no widgets is $14000. Suppose the total cost is $20000 if 500 widgets are produced.

a. Assuming a linear relationship between total cost C and number of widgets produced n, find and interpret the slope of the function $C = f(n)$.
b. Give the formula for the function $C = f(n)$.
c. What is the total cost to when 785 widgets are produced.

$(\overset{x_1}{0}, \overset{y_1}{14000}) \quad (\overset{x_2}{500}, \overset{y_2}{20000}) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{20000 - 14000}{500 - 0} = \frac{60\cancel{0}\cancel{0}}{5\cancel{0}\cancel{0}} = 12$$

$12 cost of producing 1 item

$$C(n) = 12n + 14000$$

$$C(785) = 12(785) + 14000 = \$23{,}000$$

Example 4: A certain jeweler makes a profit of \$160 when she sells 12 necklaces and \$300 when she sells 17 necklaces.

a. Assuming a linear relationship between profit P and the number of necklaces sold n, find and interpret the slope of the function $P = f(n)$.

b. Give the formula for the function $P = f(n)$.

$$(\overset{x_1}{12}, \overset{y_1}{160}) \qquad (\overset{x_2}{17}, \overset{y_2}{300})$$

$$m = \frac{300-160}{17-12} = \frac{140}{5} = 28$$

$$y - y_1 = m(x - x_1)$$

$$y - 160 = 28(x - 12)$$

$$y - 160 = 28x - 336$$

$$+160 \qquad\qquad +160$$

$$y = 28x - 176$$

$$\boxed{P(n) = 28n - 176}$$