Math 1311
Section 3.3
Modeling Data with Linear Functions

Most of the "real world" does not come with math equations attached, like price tags. You observe a particular phenomenon and obtain data points. Then you try to see if a particular type of mathematical equation gives a good description of the problem, and the function values match (or approximately match) the data collected.

Why do you want to model a situation with a mathematical equation?

- Mathematical equation can be used to find values not on the table.
- If the trends in the data are reflected in the math equation, the equation can be used to estimate what might happen in the future.

The rest of this course will be about finding appropriate models to use given a set of data.

The simplest type of mathematical function to use to model a problem is a linear function.
This is a good fit if the rate of change is constant or at least approximately constant.
In this section we will model data for which the rate of change is constant, in the next section we will look at how you handle data that is approximately linear, but not exactly.

Skills from this section:

1. Given a table of data points, use the average rate of change to decide if the situation can be modeled by a linear function.
2. If a linear model is appropriate, find a formula that fits the given data.
3. Define variables so that the linear functions are easy to interpret and the numbers are not too large by using time since a particular date, or population in thousands.
4. Graph discrete data points.
5. Comparing the graph of the discrete data with the graph of the linear model.

Example 1: Falling Objects
An object is dropped off a high building and the velocity as a function of time is displayed in the following table:

Time t in sec	0	2	5	9
Velocity V ft/sec	$\boxed{0}$	64	160	288

a. Calculate the average rate of change for each time interval:

Interval	0 - 2	2 - 5	5 - 9
AROC	32	32	32

$\frac{64-0}{2-0} = 32$ $\frac{160-64}{5-2} = 32$ $\frac{288-160}{9-5} = \frac{128}{4} = 32$

b. Is this data linear? YES, AROC is const. (32)

c. Find a formula that describes the velocity of the object as a function of the time t since it was dropped.

$y = mx + b$

AROC Initial cond.

$m = 32$

$b = 0$

$V(t) = 32t + 0$

$\boxed{V(t) = 32t}$

d. What does V(0) represent?

Initial velocity

e. What is the object's velocity 7 seconds after it was dropped?

$V(7) = 32(7) = \boxed{224 \text{ ft/sec}}$

The following table shows the position of the object as a function of the time since it was dropped. Is this data linear?

Time t in sec	0	1	2	3	4	5
Distance D In feet	0	16	64	144	256	400

Interval	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
AROC	16	48			

$\frac{16-0}{1-0} = 16$

$\frac{64-16}{2-1} = 48$

Data is NOT linear.

Example 2: Voter Registration
The following two tables show the number of registered voters in particular counties in Oklahoma as a function of the date.

Table 1

Date	2000	2001	2002	2003	2004	2005
Votes	28321	28542	29466	30381	30397	31144

$$\frac{28542-28321}{\cancel{2001-2000}} = 221$$

$1-0$

Table 2

Date	2000	2001	2002	2003	2004	2005
Votes	28321	28783	29245	29707	30169	30631

$$\frac{29466-28543}{2-1} = 924$$

Instead of using the date for our "x" variable, let's use the time in years since the year 2000.
Why?
Given our new variable x = time in years since 2000, construct new tables 1 and 2.

New Table 1

x in years since 2000	0	1	2	3	4	5
Votes	28321	28542	29466	30381	30397	31144

New Table 2

x in years since 2000	0	1	2	3	4	5
Votes	28321	28783	29245	29707	30169	30631

a. Which of these two could be modeled by a linear function?

Table 1 AROC

Interval	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
AROC	221 ≠	924			

Table 1 is NOT linear!

Table 2 AROC

Interval	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
AROC	462	462	462	462	462

b. Write the formula that represents the linear data. Linear

$$\boxed{V(x) = 462x + 28321}$$

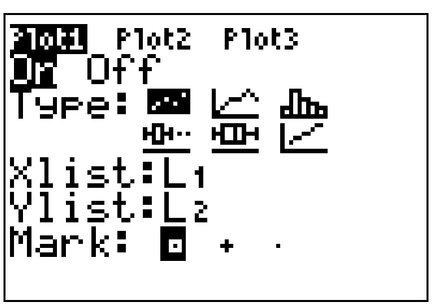
Notice that our x variable change to time since 2000 makes the formula less cumbersome. This is a standard technique in modeling situations; you "scale" the independent variables so that the functions do not end up being as messy. We could also scale the function values to be number of voters in thousands, so then our y values would be about 30 instead of 30 thousand.
Plotting a Table of Data

Graphing a set of data
We will walk through plotting the data from the first example.

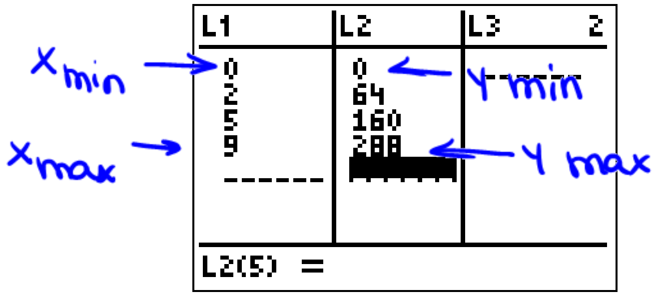
Time t in sec	0	2	5	9	L1
Velocity V ft/sec	0	64	160	288	L2

Press 

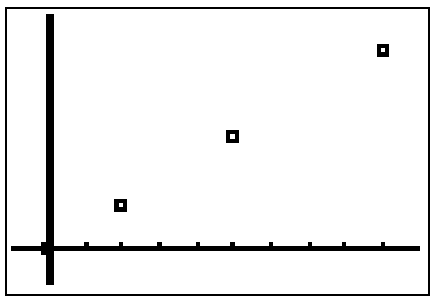
, select Plot1, turn the plot on, and choose scatterplot, set Xlist and Ylist



Press  , choose “EDIT”, and put the x-values in L1 and the y-values in L2



Press   to see the scatterplot



Example 3:

Plot each of the following tables of data. Does the plot look linear?

a.

x	1.3	2.5	3.3	4.2	5.1	L1
y	2.685	1.785	1.185	0.51	-0.165	L2

Linear

b.

x	0.3	1.3	2.2	3.3	4.1	L1
y	5.6	2	0.92	0.77	0.51	L2

NOT Linear

Key Idea
To test if a table of data is linear, calculate the average rate of change between each consecutive pair of points.

- If the rate of change is constant, the data represents a linear function.
- If not, then it is not a linear function.

Finding a Formula for a Table that is Linear

The average rate of change found when determining that the table is linear is the slope of the line.

Use this slope and any one point from the table, write the equation using the point-slope form, and then solve for "y" to get the function equation.

$$y - y_1 = m(x - x_1)$$