Math 1311
Section 3.5
Systems of Equations

Many times physical situations are best described by a system of two linear equations with two unknowns. We are going to look at a few of these situations, and solve them both algebraically and graphically.

Example 1: Gravel
We have $1800 to spend on the repair of a long gravel driveway. We want to make the repair using a mix of coarse gravel priced at $28 a ton and fine gravel priced at $32 a ton. To make a good driving surface, we need 3 times as much fine gravel as coarse gravel. How much of each type of gravel should we purchase?

Let c = amount of coarse gravel
f = amount of fine gravel

$$\begin{cases} f = \boxed{3c} \\ 28c + 32f = 1800 \end{cases}$$

$$28c + 32(3c) = 1800$$
$$28c + 96c = 1800$$
$$\frac{124c}{124} = \frac{1800}{124}$$
$$c = 14.52$$

$$f = 3c = 3(14.52)$$
$$= 43.55$$

Need 14.52 tons of coarse gravel
43.55 tons of fine gravel

Methods for solving systems:

Graphically:

1. Solve each equation for the same variable. y
2. Graph the lines that result.
3. Find the intersection point of the two lines.

Example 2: Solve the following system of two equations graphically.

$$3x + 4y = 6$$
$$2x - 6y = 5$$

$$\boxed{\begin{array}{l} x = 2.15 \\ y = -0.12 \end{array}}$$

$3x + 4y = 6$

$-3x \quad -3x$

$\frac{4y}{4} = \frac{-3x + 6}{4}$

$y = \underbrace{\frac{-3x+6}{4}}_{Y_1}$

$2x - 6y = 5$

$-2x \quad -2x$

$\frac{-6y}{-6} = \frac{-2x+5}{-6}$

$y = \frac{-2x+5}{-6} = \underbrace{\frac{2x-5}{6}}_{Y_2}$

Algebraically:

1. Solve one equation for one of the variables.
2. Replace that variable in the OTHER equation by the expression it is equal to from part 1.
3. Solve this equation.
4. Use the answer in part 3 to find the value for the second variable.

Example 3: Solve the following system of two equations algebraically.

$$3x - y = 5$$
$$2x + y = 0 \rightarrow y = -2x$$

$3x - (-2x) = 5$

$3x + 2x = 5$

$5x = 5$

$x = 1$

$y = -2x = -2(1) = -2$

$$\boxed{\begin{array}{l} x = 1 \\ y = -2 \end{array}}$$

$3x - y = 5$

$-3x \quad -3x$

$(-1) - y = (-3x + 5)(-1)$

$y = \underbrace{3x - 5}_{Y_1}$

$2x + y = 0$

$-2x \quad -2x$

$y = \underbrace{-2x}_{Y_2}$

$$\boxed{\begin{array}{l} x = 1 \\ y = -2 \end{array}}$$

Example 4: You have $36 dollars to spend on refreshments for a party. Bags of chips cost $2.00 and sodas cost 50 cents each. You need to buy 5 times as many sodas as bags of chips. How many of each can you afford to buy on your budget?

c = # of bags of chips

s = # of sodas

50¢ = $.5

$$\begin{cases} s = 5c \\ 2c + .5s = 36 \end{cases}$$

$s = 5c = 5(8) = 40$

$2c + .5(5c) = 36$

$2c + 2.5c = 36$

$\frac{4.5c}{4.5} = \frac{36}{4.5}$

$c = 8$

$$\boxed{\begin{array}{l} \text{8 bags of chips} \\ \text{40 sodas} \end{array}}$$

Example 4: Coins

A bag contains 32 coins, some nickels and some quarters. If there is a total of $4.60 in the bag, how many of each type of coin does the bag contain?

n = # of nickels
q = # of quarters

$$\begin{cases} n + q = 32 \\ 5n + 25q = 460 \end{cases} \quad \text{OR} \quad \begin{cases} n + q = 32 \\ .05n + .25q = 4.60 \end{cases}$$

$$n + q = 32$$
$$-q \quad -q$$
$$n = \underbrace{32 - q}_{Y_1}$$

$$5n + 25q = 460$$
$$-25q \quad -25q$$
$$\frac{5n}{5} = \frac{460 - 25q}{5}$$
$$n = \underbrace{92 - 5q}_{Y_2}$$

$q = x = 15$
$n = y = 17$

$$\boxed{\begin{array}{l} \text{15 quarters} \\ \text{17 nickels} \end{array}}$$