Math 1311
Section 4.1
Exponential Growth and Decay

A function that grows or decays by a constant percentage change over each fixed change in input is called an exponential function.

Exponents – A quick review

1. $a^0 = 1$
2. $a^{-n} = \frac{1}{a^n}$
3. $a^{\frac{1}{n}} = \sqrt[n]{a}$
4. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
5. $a^m a^n = a^{m+n}$
6. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
7. $(a^m)^n = a^{mn}$
8. $(ab)^n = a^n b^n$
9. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
10. For $b \neq 1$, $b^x = b^y$ means $x = y$.

Example 1: Simplify. Write your final answer without negative exponents.

1. $((a^2)^3)^4$
2. $\frac{a^3 b^2}{a^2 b^3}$
3. $a^3 b^2 a^4 b^{-1}$
4. $\frac{a^{-4} b^{-3}}{a^{-6} b^{-2}}$

Exponential Growth

Example 2: A petri dish contains 500 bacteria at the start of an experiment. The number of bacteria double each hour. We can calculate the number of bacteria in the dish as a function of the number of hours since the experiment started. Here is the beginning of the chart:

Time, in hours	0	1	2	3	4	5
Number of bacteria						

Let $N = N(t)$ be the number of bacteria in the dish t hours after the experiment started.

Let's use the pattern in the chart above to develop a formula for $N(t)$.

The growth factor is:

The Exponential Growth Formula: $N(t) = Pa^t, a > 1$.

P is the initial value, t is the time and a is the growth factor for each unit of time.

Exponential Decay

Example 3: Suppose we change the experiment in Example 2 by introducing an antibiotic into the petri dish. Now, the number of bacteria in the dish is cut in half each hour.

Time, in hours	0	1	2	3	4	5
Number of bacteria						

Let $N(t)$ be the number of bacteria in the petri dish t hours after the antibiotic was introduced. The pattern in the above chart suggests the following formula for $N(t)$:

The decay factor is:

The Exponential Decay Formula: $N(t) = Pa^t, a < 1$.

Example 4: **Radioactive Decay**

If there is 1 gram of heavy hydrogen in a container, then as a result of radioactive decay there will be .783 grams of heavy hydrogen in the container one year later. Suppose a container starts with 25 grams of hydrogen.

a. Find the formula for the number of grams of hydrogen in the container as a function of the time t in years.
b. How much heavy hydrogen is left after 5 years?
c. Plot the graph of the function.
d. Find the time t when $1/2$ of the hydrogen is left in the container.

Constant Proportional Change

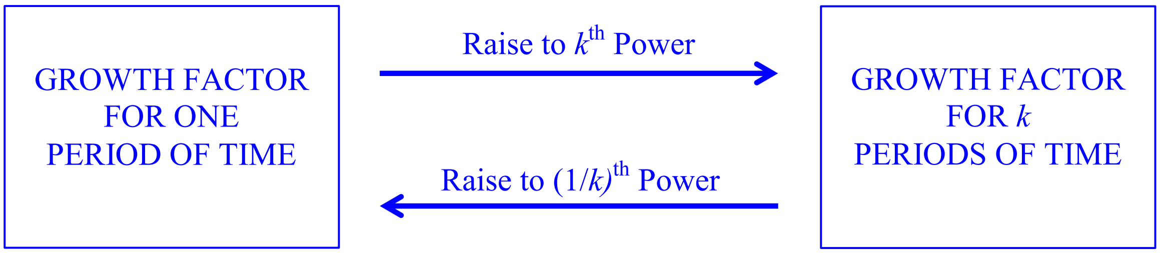
A function is exponential if it shows constant percentage (or proportional) growth or decay.
Growth: For an exponential function with discrete (yearly, monthly, etc.) percentage growth rate r as a decimal, the growth factor $\boldsymbol{a = 1 + r}$.
Decay: For an exponential function with discrete (yearly, monthly, etc.) percentage decay rate r as a decimal, the decay factor $\boldsymbol{a = 1 - r}$.

Example 5: A certain phenomenon has an initial value of 23 and grows at a rate of 6% per year. Give an exponential function which describes this phenomenon.

Example 6: Suppose the amount of pollution in a tank starts at 100 pounds and decreases by 16% per hour. Find the decay constant and the formula for the amount of pollutant in the tank in pounds as a function of time in hours. How much is left in the tank after 10 hours? 15 hours?

Growth or Decay Factor Unit Conversion
If the growth or decay factor for one period of time is a, then the growth or decay factor for k periods of time is given by $A = a^k$.

Here is the conversion diagram:



What does this mean in practical terms?
Example 7: Census data is collected every 10 years. Suppose the census data shows that the population increases by 23% per decade. What is the yearly growth factor?

Example 8: Terry deposits $10,000 in an account with a 0.75% monthly interest rate. What is the yearly growth factor?

Find the amount in the account after 10 years.