Math 1311
Section 4.2
Constant Percentage Change

We continue to work with exponential functions having formula $f(x) = Pa^x$, where P is the initial amount of output and a is the growth or decay factor.

- For an exponential function with discrete (yearly, monthly, etc.) percentage growth rate r as a decimal, the growth factor is $a = 1 + r$.
- For an exponential function with discrete percentage decay rate r as a decimal, the decay factor is $a = 1 - r$.
- The percentage growth/decay ~~factor~~ rate is NOT the same as the growth/decay factor!

Example 1: A Savings Account

You initially invest \$500 in a savings account that pays a yearly interest rate of 4%.

a. Write a formula for an exponential function giving the balance in your account as a function of time since your initial investment.
b. What monthly interest rate best represents this account? Round your answer to three decimal places.
c. Calculate the decade growth factor.
d. Use the formula you found in part a to determine how long it will take for the account to reach \$740.

a) $r = 4\% \rightarrow .04$

$a = 1 + .04 = 1.04$

$I = Pa^t$

$I = 500(1.04)^t$

b) r yearly = .04

yearly growth factor = 1.04

monthly growth factor = $(1.04)^{1/12}$

= 1.003

r monthly = 1.003 − 1

= .003

.3%

c) yearly growth factor = 1.04

decade growth factor = $(1.04)^{10} = 1.48$

d) $\underbrace{500(1.04)^t}_{Y_1} = \underbrace{740}_{Y_2}$

t = 10 years

Example 2: At age 25 you start to work for a company and are offered two rather fanciful retirement options.

Option 1: When you retire, you will be paid a lump sum of $25000 for each year of service.

Option 2: When you start to work, the company will deposit $10000 into an account that pays 1% per month. When you retire, the account will be closed and the balance given to you.

a. Which retirement option is more favorable to you if you retire at age 65?
b. Which retirement option is more favorable if you retire at age 55?

(a) 40 years of service

Opt.1 25000 (40) $1,000,000

✓ Opt.2 1% per month $r = .01$

monthly growth factor $= 1 + .01 = 1.01$

yearly growth factor $= (1.01)^{12} = 1.1268$

$A(t) = 10000\,(1.1268)^{40}$ $1,185,423.5

(b) 30 years of service

Opt.1. 25000(30) = $750,000 Op.2 $10000\,(1.1268)^{30}$ = $359,256.93

Example 3: The half-life of a certain radioactive substance is 14.5 hours.

a. Find the hourly decay factor for this substance.
b. What is the constant percentage change for this substance?

Ex: 100gr —14.5 hours→ 50gr

$a = 1 - r$ $a + r = 1$

$r = 1 - a$

50% or .5 change per 14.5 hours

a) Decay factor for 14.5 hours $= 1 - .5 = .5$

Hourly decay factor $(.5)^{1/14.5} = .9533$

b) $1 - .9533 = .0467$

$.0467 \times 100 = 4.67\%$ change per hour