Math 1311
Section 4.3
Modeling Exponential Data

Given a table of data for a particular situation, how do we tell if the data is appropriately modeled by an exponential function?
From Section 4.1 we learned that a function was exponential if the function value was MULTIPLIED by the growth or decay factor each time the "x" value was increased by 1.

$$New\ Value\ =\ Growth\ Factor\ \times\ Old\ Value$$

Or, in other words,

$$\boxed{\frac{New\ Value}{Old\ Value}\ =\ Growth\ Factor}$$

This means that, for evenly spaced data, the successive QUOTIENTS should be the same in an exponential function.

Example 1: You invest money into a savings account that pays a fixed monthly interest rate. The following table shows the account balance over the first 5 months.

Time in months	**Balance**
0	1750.00
1	1771.00
2	1792.25
3	1813.76
4	1835.52
5	1857.55

+1
+1
+1
+1
+1

$N(t) = P a^t$

Show that the data is exponential by calculating the quotients.

Time increment	From 0 to 1	From 1 to 2	From 2 to 3	From 3 to 4	From 4 to 5
Ratios of B	$\frac{1771}{1750} = 1.012$	$\frac{1792.25}{1771} = 1.012$	$\frac{1813.76}{1792.25} = 1.012$	$\frac{1835.52}{1813.76} = 1.012$	$\frac{1857.55}{1835.52} = 1.012$

What is the monthly growth factor?

1.012

growth factor > 1

$0 <$ decay factor < 1

What is the initial value?

1750

To find a formula for the amount in this account as a function of time in years, we need the yearly growth factor. Calculate this.

$$YGF = (MGF)^{12}$$

$$YGF = (1.012)^{12} = 1.154$$

Write an exponential function which models this situation and explain the variables you use.

$$N(t) = 1750(1.154)^t$$

t = # of years since the opening of account

Find the amount in the account 15 years after the account was opened.

$$N(15) = 1750(1.154)^{15}$$ \$15,001.17

How long does it take for the amount in the account to double? How long does it take to double again?

$$1750(2) = 3500$$

$$\underbrace{1750(1.154)^t}_{Y_1} = \underbrace{3500}_{Y_2}$$ $t \approx 4.8$ years

Window: [0, 15] × [0, 5000]

$$3500(2) = 7000$$

$$\underbrace{1750(1.154)^t}_{Y_1} = \underbrace{7000}_{Y_2}$$ $t \approx 9.7$ years

$N(t) = Pa^t$

Example 2: Show that this data is exponential and find the formula for the exponential model.

Time, t	0	1	2	3	4	5
F(t)	3.80	3.95	4.11	4.27	4.45	4.62

$$\boxed{F(t) = 3.80(1.04)^t}$$

Time increment	From 0 to 1	From 1 to 2	From 2 to 3	From 3 to 4	From 4 to 5
Ratios of F	$\frac{3.95}{3.80} = 1.04$	$\frac{4.11}{3.95} = 1.04$	$\frac{4.27}{4.11} = 1.04$	$\frac{4.45}{4.27} = 1.04$	$\frac{4.62}{4.45} = 1.04$

Let's look at the difference between a linear model and an exponential model.

Example 3: Which of the following is linear and which is exponential?

Table 1

+1 +1 +1

Time t	0	1	2	3
F(t)	7.75	8.99	10.43	12.10

Table 2

Time t	0	1	2	3
G(t)	7.72	8.99	10.26	11.53

Calculate Differences:

Table 1: $8.99 - 7.75 = 1.24$; $10.43 - 8.99 = 1.44$ Not linear

Table 2: $8.99 - 7.72 = 1.27$; $10.27 - 8.99 = 1.27$; $11.53 - 10.26 = 1.27$

is Linear

Calculate Quotients:

Table 1: $\frac{8.99}{7.75} = 1.16$ $\frac{10.43}{8.99} = 1.16$ $\frac{12.10}{10.43} = 1.16$

is exponential

Notice that the LINEAR table has the same DIFFERENCES and the EXPONENTIAL table has the same QUOTIENTS

Example 4: The following table shows the amount of heavy hydrogen N in a container as a function of the time in years.

Time in years	**N, grams**
0	100
1	78.3
2	61.31
3	48.01
4	37.59
5	29.43

$\frac{78.3}{100} = .783$

$\frac{61.31}{78.3} = .783$

$\frac{48.01}{61.31} = .783$

$\frac{37.59}{48.01} = .783$

$\frac{29.43}{37.59} = .783$

Show that this is exponential data and find an exponential model. (Be sure to define variables with units!)

t = time in years

$N(t)$ = grams

$N(t) = Pa^t$

$\boxed{N(t) = 100(.783)^t}$

What is the percentage decay rate for each year?

$D.F = .783$

$1 - D.F = 1 - .783 = .217$

$.217 \times 100 = 21.7\%$

Use functional notation to express the amount of hydrogen in the container 10 years after the experiment begins.

$N(10) = 100(.783)^{10} = \boxed{8.66 \text{ grams}}$

What is the half-life of heavy hydrogen?

$100/2 = 50$

$\underbrace{100(.783)^t}_{Y_1} = \underbrace{50}_{Y_2}$

$\boxed{t = 2.83 \text{ years}}$

Note to show that NOT equally spaced data is exponential you need to show for any two data points (x_1, y_1) and (x_2, y_2) the following value is the same

$$\left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

Example 5: A freezer maintains a constant temperature of 6 degrees Fahrenheit. An ice tray is filled with tap water and placed in the freezer to make ice. The difference between the temperature of the water and the temperature of the freezer was sampled each minute and recorded in the table that is given. Let's first see if this data appears to be exponential.

+1 +3 +1

Time, t	0	1	4	5
Temp difference	69.0	66.3	58.8	56.5

$$\left(\frac{66.3}{69.0}\right)^{\frac{1}{1-0}} = .96 \qquad \left(\frac{58.8}{66.3}\right)^{\frac{1}{4-1}} = .96 \qquad \left(\frac{56.5}{58.8}\right)^{\frac{1}{5-4}} = .96$$

Find an exponential model for the temperature difference between the freezer and the water.

$$D(t) = 69(.96)^t$$

Find a model for the temperature of the water as a function of time in minutes.

$$T(t) - 6 = 69(.96)^t$$

$$T(t) = 6 + 69(.96)^t$$

When will the water temperature reach 32 degrees?

$$\underbrace{6 + 69(.96)^t}_{Y_1} = \underbrace{32}_{Y_2}$$

$t = 24$ minutes