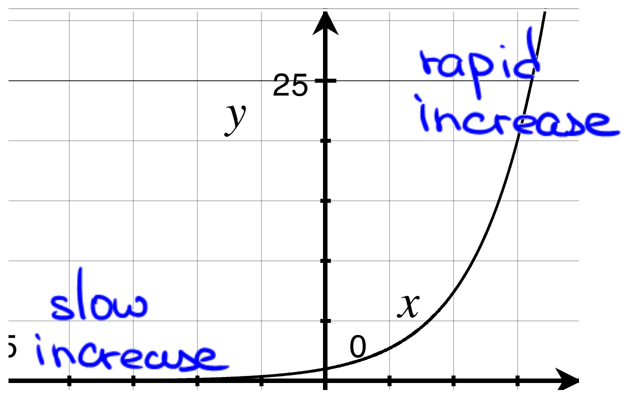
Math 1311
Section 4.4
Modeling Nearly Exponential Data: Exponential Regression

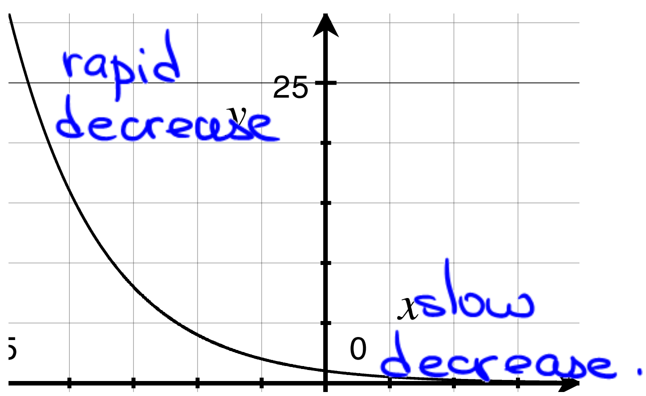
When we looked at linear data, we saw that often the data were not perfectly linear, but we could still model the data using a linear function.
Similarly, sometimes data is not exactly exponential, but we can still use an exponential function to model the data.

Graphs of Exponential Function
The graph of an exponential function $f(x) = Pa^x$ has one of two forms (depending on whether it is exponential growth or decay).



Always increasing

$a > 1$: exponential growth



Always decreasing

$a < 1$: exponential decay

The graphs of exponential functions are always concave up and either always increasing or always decreasing.

If the scatter plot of a data set shows either of these “shapes”, then an exponential regression model is likely appropriate.

xmin xmax

Example 1: This data gives US population in millions from 1800 to 1860:

x	0	10	20	30	40	50	60
Date	1800	1810	1820	1830	1840	1850	1860
Population	5.31	7.24	9.64	12.87	17.07	23.19	31.44

Graph the data and answer this question: Is it reasonable to approximate the data with an exponential model?

YES

Use the exponential regression feature of the statistical calculator to find the initial value and the growth factor of the linear regression model for this data. Then write an exponential function that models this data.

$$P(t) = 5.34(1.03)^t$$

VARS → Y-VARS → Enter

Use the model to approximate the population in 1870.

$t = 70$

$P(70) = 41.82$ millions of people

Use the model to determine the approximate year when the population crossed 50 million people

$$\underbrace{5.34(1.03)^t}_{Y_1} = \underbrace{50}_{Y_2} \qquad t = 76$$

in 1876

Example 2: Use the data given in the table below to find an exponential regression model that fits the data.

x	4.2	7.9	10.8	15.5	20.2
y	7.5	8.1	8.5	10.2	12.3

$f(x) = 6.35(1.03)^x$

Use the model to find the value when $x = 12$.

$f(12) = 9.25$

with rounding $f(12) = 9.05$

Use the model to find the value of x for which the model equals 9.

$\underbrace{6.35(1.03)^x}_{Y_1} = \underbrace{9}_{Y_2}$

$x = 11.13$ (rounded $x = 11.8$)

Example 3: The table below shows the number of cell phone subscribers (in millions) in the US at the end of the given year.

x	1	2	3	4	5
Year	2001	2002	2003	2004	2005
Subscribers	128.4	140.8	158.7	182.1	207.9

Plot the data points. Is it reasonable to approximate the data using an exponential model?

x = # of years since 2000

YES

Use exponential regression to construct an exponential model for the data.

$$N(x) = 111.71(1.13)^x$$

Graph the exponential regression model with the data points.

What was the yearly percentage growth rate from the end of 2001 through the end of 2005?

growth factor $= 1.13$

$1.13 - 1 = .13 \qquad .13 \times 100 = \boxed{13\%}$

Would this model support this statement (made in 2005): "By the end of 2007, there will be 250 million cell phone subscribers in the US."

$N(7) = 262.8$ millions

Using this model, in what year would you expect cell phone subscribership to reach 275 million?

$$\underbrace{111.71(1.13)^x}_{Y_1} = \underbrace{275}_{Y_2} \qquad x = 7.37$$

≈ 7.37 years