Math 1311
Section 4.5
Logarithmic Functions

In the last couple of sections, we looked at exponential functions. With an exponential function like $N(t) = 2^t$, a small increase in t results in a large increase in $N(t)$.

In this section, we will look at logarithms. One of the characteristics of a logarithmic function is that a large increase in the independent variable results in a fairly small increase in the function value.

We will first consider these functions by looking at a familiar example.

Example 1: The Richter scale
The magnitude of earthquakes is measured on the Richter scale, a scale created by Charles Richter in 1935.

Moderate quake – 5.3
Strong quake – 6.3
Strongest quake recorded – 8.9

The rule is that an increase in 1 point on the Richter scale indicates a quake that is 10 times more powerful.

a) Comparing a 5.3 and a 6.3 –

b) The 1811 quake in New Madrid, Missouri with a magnitude of 8.8.
San Francisco 1989 – a quake of magnitude 7.1
How much more powerful was the New Madrid quake than the San Francisco quake?

c) If an earthquake 100 times more powerful than the San Francisco quake occurred, what would its Richter scale measurement be?

d) In 2004 a quake which was 80 times more powerful than the San Francisco quake happened in Indonesia. What was the Richter scale measurement for the 2004 quake?

The Common Logarithm

Notice that the solution to the last part of Example 1 asks us to solve the equation $10^x = 80$ for x. We can use the crossing graph method to solve this on our calculator. However, if we need to solve many equations like this one, we might want a more efficient way of doing so. This efficient solution is accomplished by the common logarithm.

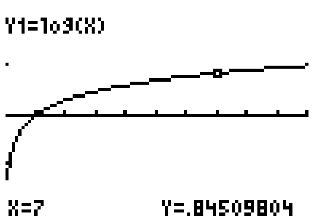
Definition of the common logarithm: $y = \log x$ is equivalent to the equation $10^y = x$.

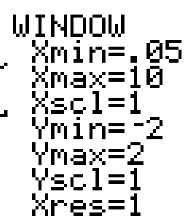
The graphing calculator has a key to calculate log x, so the solution to the equation $10^x = 80$ is the number $x = \log 80$ which we can find with a few keystrokes.

Example 2:

$$\log 1 = 0 \; because \; 10^0 = 1$$
$$\log 10 = 1 \; because \; 10^1 = 10$$
$$\log 100 = 2 \; because \; 10^2 = 100$$
$$\log 1000 = 3 \; because \; 10^3 = 1000$$
$$\log 0.01 = -2 \; because \; 10^{-2} = 0.01$$
$$\log 5 = 0.7 \; because \; 10^{0.7} = 5 \text{ (Check this using your calculator)}$$

The graph of the common logarithm





Basic Facts for the Common Logarithm

1. $\log x$ is the power of 10 that gives x.
2. If you multiply a number by 10^t, you increase it's logarithm by t units.
3. The function $f(x) = \log x$ increases slowly and is concave down. This means it increases at a decreasing rate as x gets bigger.

The logarithm is the inverse of the exponential function.

Example 3:

a) $\log 1000000$

b) $\log 10^{3.4}$

c) $10^t = 60$

d) $\log(2x + 5) = 1.2$

e) $3.2 \log(3x + 8) - 2 = 3.76$

Example 4: Zoologists have studied the daily rate of gain in weight G as a function of daily milk- energy intake M during the first month of life in several hoofed mammal species. The model is

$$G = 0.067 + 0.052 \log M$$

a) Draw a graph if G versus M including values of M up to .4 units.

b) If the daily milk-energy intake M is 0.3 units, what is the daily rate of gain in weight?

c) A zookeeper wants to bottle-feed an elk calf so as to maintain a daily rate of gain in weight G of 0.03 units. What must the daily milk-energy intake be?

d) What does the shape of the graph say about the efficiency of higher levels of milk-energy intake?

The Natural Logarithm

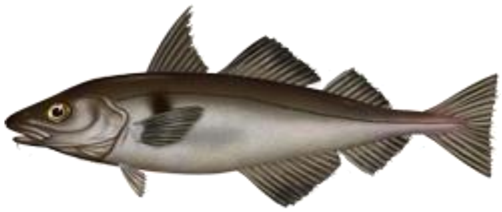
Definition of the natural logarithm: $y = \ln x$ is equivalent to the equation $e^y = x$.

Example 5:

$$\ln 1 = 0 \; because \; e^0 = 1$$
$$\ln e = 1 \; because \; e^1 = e$$
$$\ln e^2 = 2 \; because \; 2 \; is \; the \; power \; of \; e \; that \; gives \; e^2$$
$$\ln e^t = t \; because \; t \; is \; the \; power \; of \; e \; that \; gives \; e^t$$

Example 6: Solve for x: $\ln(2x + 5) = 1.2$

Example 7: Age of Haddock
The age T, in years, of a haddock can be thought of as a function of its length L, in centimeters. One common model uses the natural logarithm: $T = 19 - 5\ln(53 - L)$



a) Draw a graph of age versus length. Include lengths between 25 and 50 centimeters.

b) Express using functional notation the age of a haddock that is 35 centimeters long, and then calculate that value.

c) Low long is a haddock that is 10 years old?