Math 1311
Section 4.5
Logarithmic Functions

In the last couple of sections, we looked at exponential functions. With an exponential function like $N(t) = 2^t$, a small increase in t results in a large increase in $N(t)$.

In this section, we will look at logarithms. One of the characteristics of a logarithmic function is that a large increase in the independent variable results in a fairly small increase in the function value.

We will first consider these functions by looking at a familiar example.

Example 1: The Richter scale
The magnitude of earthquakes is measured on the Richter scale, a scale created by Charles Richter in 1935.

Moderate quake – 5.3
Strong quake – 6.3
Strongest quake recorded – 8.9

The rule is that an increase in 1 point on the Richter scale indicates a quake that is 10 times more powerful.

a) Comparing a 5.3 and a 6.3 –

$6.3 - 5.3 = 1$

6.3 is 10 times more powerful than 5.3.

b) The 1811 quake in New Madrid, Missouri with a magnitude of 8.8.
San Francisco 1989 – a quake of magnitude 7.1
How much more powerful was the New Madrid quake than the San Francisco quake?

$8.8 - 7.1 = 1.7$

$10^{1.7} = 50$ 50 times more powerful

c) If an earthquake 100 times more powerful than the San Francisco quake occurred, what would its Richter scale measurement be?

$10^2 = 100$

$7.1 + 2 = 9.1$

d) In 2004 a quake which was 80 times more powerful than the San Francisco quake happened in Indonesia. What was the Richter scale measurement for the 2004 quake?

$\underbrace{10^t}_{Y_1} = \underbrace{80}_{Y_2}$

$t = 1.9$

$7.1 + 1.9 = 9.0$

$\log_b a$ ← argument, ← base

The Common Logarithm

Notice that the solution to the last part of Example 1 asks us to solve the equation $10^x = 80$ for x. We can use the crossing graph method to solve this on our calculator. However, if we need to solve many equations like this one, we might want a more efficient way of doing so. This efficient solution is accomplished by the common logarithm.

Definition of the common logarithm: $y = \log_{10} x$ is equivalent to the equation $10^y = x$.

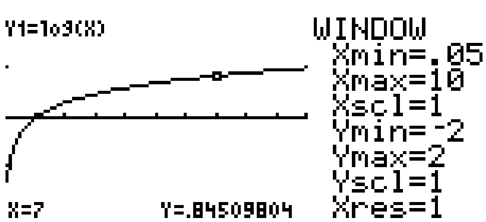
The graphing calculator has a key to calculate log x, so the solution to the equation $10^x = 80$ is the number $x = \log 80$ which we can find with a few keystrokes.

Example 2:

$$\log 1 = 0 \; because \; 10^0 = 1$$
$$\log 10 = 1 \; because \; 10^1 = 10$$
$$\log 100 = 2 \; because \; 10^2 = 100$$
$$\log 1000 = 3 \; because \; 10^3 = 1000$$
$$\log 0.01 = -2 \; because \; 10^{-2} = 0.01$$
$$\log 5 = 0.7 \; because \; 10^{0.7} = 5 \text{ (Check this using your calculator)}$$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = .01$

The graph of the common logarithm



Basic Facts for the Common Logarithm

1. $\log x$ is the power of 10 that gives x.
2. If you multiply a number by 10^t, you increase it's logarithm by t units.
3. The function $f(x) = \log x$ increases slowly and is concave down. This means it increases at a decreasing rate as x gets bigger.

The logarithm is the inverse of the exponential function.

Example 3:

a) $\log 1000000$ $= 6$

$10^t = 1{,}000{,}000$

$10^6 = 1000000$

b) $\log 10^{3.4}$ $= 3.4$

$10^t = 10^{3.4}$

c) $10^t = 60$

$\log 60$

$t = 1.78$

d) $\log(2x+5) = 1.2$

$10^{1.2} = 2x+5$

$-5 \qquad -5$

$\dfrac{10^{1.2}-5}{2} = \dfrac{2x}{2}$

$x = \dfrac{10^{1.2}-5}{2}$ $\qquad x = 5.42$

e) $3.2\log(3x+8) - 2 = 3.76$

$+2 \quad +2$

$\dfrac{3.2\log(3x+8)}{3.2} = \dfrac{5.76}{3.2}$

$\log(3x+8) = 1.8$

$10^{1.8} = 3x+8$

$-8 \qquad -8$

$\dfrac{10^{1.8}-8}{3} = \dfrac{3x}{3}$

$x = \dfrac{10^{1.8}-8}{3}$

$\boxed{x = 18.37}$

Example 4: Zoologists have studied the daily rate of gain in weight G as a function of daily milk- energy intake M during the first month of life in several hoofed mammal species. The model is

$$G = 0.067 + 0.052 \log M$$

a) Draw a graph if G versus M including values of M up to .4 units.

$x_{min} = 0$

$x_{max} = .4$

b) If the daily milk-energy intake M is 0.3 units, what is the daily rate of gain in weight?

$$G(0.3) = 0.067 + 0.052 \log 0.3$$
$$= .04$$

c) A zookeeper wants to bottle-feed an elk calf so as to maintain a daily rate of gain in weight G of 0.03 units. What must the daily milk-energy intake be?

$$\underbrace{0.067 + 0.052 \log M}_{Y_1} = \underbrace{0.03}_{Y_2}$$

$$M = .19 \text{ units}$$

d) What does the shape of the graph say about the efficiency of higher levels of milk-energy intake?

The Natural Logarithm $e \approx 2.7$

Definition of the natural logarithm: $y = \ln x$ is equivalent to the equation $e^y = x$.

Example 5:

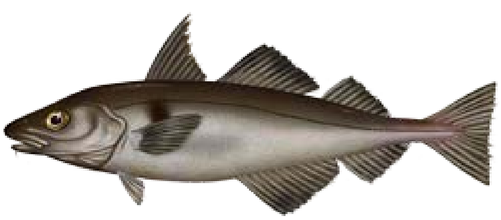
$$\ln 1 = 0 \; because \; e^0 = 1$$
$$\ln e = 1 \; because \; e^1 = e$$
$$\ln e^2 = 2 \; because \; 2 \; is \; the \; power \; of \; e \; that \; gives \; e^2$$
$$\ln e^t = t \; because \; t \; is \; the \; power \; of \; e \; that \; gives \; e^t$$

Example 6: Solve for x: $\ln(2x + 5) = 1.2$

$$e^{1.2} = 2x + 5$$
$$-5 \qquad -5$$

$$\frac{e^{1.2} - 5}{2} = \frac{2x}{2} \qquad x = \frac{(e^{1.2} - 5)}{2} \approx -.84$$

Example 7: Age of Haddock
The age T, in years, of a haddock can be thought of as a function of its length L, in centimeters. One common model uses the natural logarithm: $T = 19 - 5\ln(53 - L)$



Xmin Xmax

a) Draw a graph of age versus length. Include lengths between 25 and 50 centimeters.

b) Express using functional notation the age of a haddock that is 35 centimeters long, and then calculate that value.

$T(35) = 4.55$

c) Low long is a haddock that is 10 years old?

$$\underbrace{19 - 5\ln(53 - L)}_{Y_1} = \underbrace{10}_{Y_2} \qquad L = 46.95 \text{ cm}$$