Math 1311 Section 5.2 Power Functions

Recall, exponential functions of the form $f(x) = Pa^x$ has a fixed base *a*, and the exponent varies. For a power function this is reversed. There is a fixed exponent, and the base varies.

Power Functions

For a power function $f(x) = cx^k$

- *k* is called the power and it is the most significant part of a power function.
- The coefficient c is equal to f(1).
- If k is positive, then f is increasing; larger, positive values of k cause f to increase more rapidly.
- If k is negative, then f decreases toward zero; negative values of k that are larger in size cause f to decrease more rapidly.

Graphs of power functions



Example 1: When a rock is dropped from a tall structure, it will fall $D = 16t^2$ feet in t seconds.

- a. Make a graph that shows the distance the rock falls versus time if the building is 70 feet tall.
- b. How long does it take the rock to strike the ground?

Homogeneity Property of Power Functions

What happens to a power function when you double the variable? Triple the variable?

Example 2: The area A of a square with side length s is equal to s^2 . Calculate the area of a square if the lengths of the sides are

a. Doubled.

- b. Tripled.
- c. Quadrupled

- **Example 3:** The volume V of a cube with side length s is equal to s^3 . Calculate the volume of a cube if the lengths of the sides are
 - a. Doubled.

b. Tripled.

c. Quadrupled

General Rule:

For a power function $f(x) = cx^k$, if x is increased by a factor of t, then f is increased by a factor of t^k .

Example 4: The speed at which certain animals run is a power function of their stride length, and the power is k = 1.7. If one animal has a stride length three times as long as another, how much faster does it run?

Example 5: Let $f(x) = cx^{2.53}$. By what factor must x be increased in order to triple the value of f?

Example 6: Let $f(x) = cx^{1.47}$. If x is doubled in value, by what factor would f be increased?

Example 7: Let $f(x) = cx^k$. Suppose f(6.6) is 6.2 times as large as f(x) = 1.76. What is the value of k?

Example 8: Let $f(x) = cx^{-1.32}$ and suppose f(5) = 11. Find the value of c.

Example 9: A biologist has determined that the weight of a certain fish is a power function of its length. He also has determined that when the length doubles, the weight increases by a factor of 7.4. What is the power k?

Comparing Exponential and Power Functions

Example 10: Let's compare $f(x) = 2^x$ and $g(x) = x^2$.



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Conclusion: Over a sufficiently large horizontal span, an exponential function (with base larger than 1) will increase much more rapidly than a power function.