Math 1311
Section 5.2
Power Functions

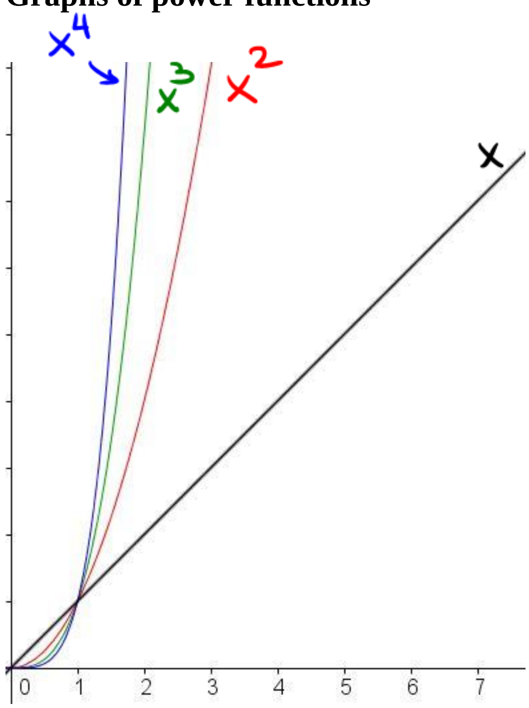
Recall, exponential functions of the form $f(x) = Pa^x$ has a fixed base a, and the exponent varies. For a power function this is reversed. There is a fixed exponent, and the base varies.

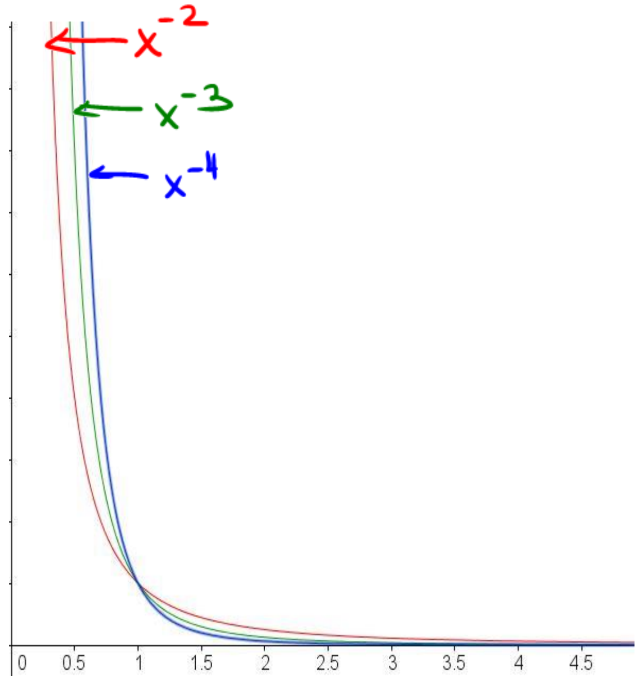
Power Functions

For a power function $f(x) = cx^k$

- k is called the power and it is the most significant part of a power function.
- The coefficient c is equal to $f(1)$. $f(1) = c(1)^k = c$
- If k is positive, then f is increasing; larger, positive values of k cause f to increase more rapidly.
- If k is negative, then f decreases toward zero; negative values of k that are larger in size cause f to decrease more rapidly.

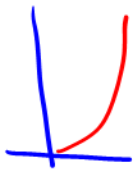
Graphs of power functions





Example 1: When a rock is dropped from a tall structure, it will fall $D = 16t^2$ feet in t seconds.

a. Make a graph that shows the distance the rock falls versus time if the building is 70 feet tall.



$[0,5] \times [0,100]$

b. How long does it take the rock to strike the ground?

$\underbrace{16t^2}_{Y_1} = \underbrace{70}_{Y_2}$

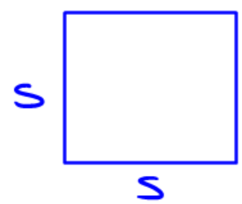
$\boxed{\text{2.09 seconds}}$

Homogeneity Property of Power Functions

What happens to a power function when you double the variable? Triple the variable?

Example 2: The area A of a square with side length s is equal to s^2. Calculate the area of a square if the lengths of the sides are

a. Doubled.



$A = s^2$

$(2s)^2 = 4s^2$

b. Tripled.

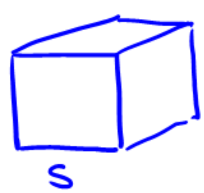
$(3s)^2 = 9s^2$

c. Quadrupled

$(4s)^2 = 16s^2$

Example 3: The volume V of a cube with side length s is equal to s^3. Calculate the volume of a cube if the lengths of the sides are

a. Doubled.



$(2s)^3 = (2)^3 \cdot s^3 = 8s^3$

b. Tripled.

$(3s)^3 = (3)^3 \cdot s^3 = 27s^3$

c. Quadrupled

$(4s)^3 = 4^3 \cdot s^3 = 64s^3$

General Rule:
For a power function $f(x) = cx^k$, if x is increased by a factor of t, then f is increased by a factor of t^k.

Example 4: The speed at which certain animals run is a power function of their stride length, and the power is $k = 1.7$. If one animal has a stride length three times as long as another, how much faster does it run?

$f(s) = cs^{1.7}$ $\quad 3^{1.7} = 6.47$

Example 5: Let $f(x) = cx^{2.53}$. By what factor must x be increased in order to triple the value of f?

$f_{old} = cx^{2.53}$

$f_{new} = c(kx)^{2.53}$

$\frac{\cancel{c}(kx)^{2.53}}{x^{2.53}} = \frac{3\cancel{c}x^{2.53}}{x^{2.53}}$ $\quad \frac{k^{2.53} \cdot \cancel{x^{2.53}}}{\cancel{x^{2.53}}} = 3$

Y_1 $\underbrace{k^{2.53}} = \underbrace{3}$, Y_2 $\quad \boxed{k = 1.54}$

Example 6: Let $f(x) = cx^{1.47}$. If x is doubled in value, by what factor would f be increased?

$2^{1.47} = 2.77$

Example 7: Let $f(x) = cx^k$. Suppose $f(6.6)$ is 6.2 times as large as ~~$f(x) = 1.76$~~ $f(1.76)$. What is the value of k?

$\frac{6.6}{1.76} = 3.75$ $\quad \underbrace{(3.75)^k}_{Y_1} = \underbrace{6.2}_{Y_2}$ $\quad k = 1.38$

$f(x) = cx^{1.38}$

$f(x) = cx^k$

Example 8: Let $f(x) = cx^{-1.32}$ and suppose $f(5) = 11$. Find the value of c.

$$\frac{c(5)^{-1.32}}{5^{-1.32}} = \frac{11}{5^{-1.32}} \qquad c = \frac{11}{5^{-1.32}} = 92.05$$

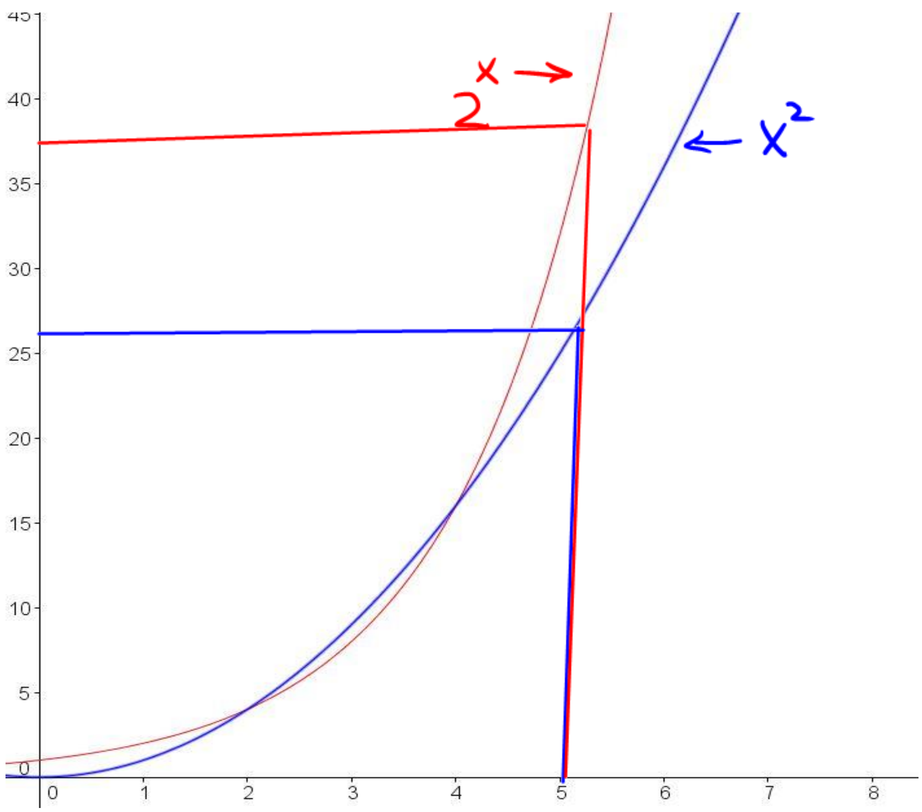
$$f(x) = 92.05x^{-1.32}$$

Example 9: A biologist has determined that the weight of a certain fish is a power function of its length. He also has determined that when the length doubles, the weight increases by a factor of 7.4. What is the power k? $f(x) = cx^k$

$$\underbrace{2^k}_{Y_1} = \underbrace{7.4}_{Y_2} \qquad k = 2.89$$

Comparing Exponential and Power Functions

Example 10: Let's compare $f(x) = 2^x$ and $g(x) = x^2$.



Conclusion: Over a sufficiently large horizontal span, an exponential function (with base larger than 1) will increase much more rapidly than a power function.