Math 1311
Section 5.4
Combining and Decomposing Functions

In this section we look more carefully at how functions are combined to make new ones and at the meaning of the individual pieces.

The simplest way of combining functions are sums, products, differences, and quotients.

Example 1: To join a book club you pay an initial fee and then a fixed price each month for a book. The total cost in dollars of joining the club and buying n books is given by $C = 70 + 25n$.

a. What is the initial fee?

$70

b. What is the cost of each book after you are a club member?

$25

Example 2: If a skydiver jumps from an airplane, his velocity , in feet per second, starts at 0 and increases toward terminal velocity. An average-sized man has a terminal velocity T of about 176 feet per second. The difference $D = T - v$ is an exponential function of time.

a. What is the initial value of D?

$T = 176$

$v(0) = 0$

$D(0) = T(0) - v(0) = 176 - 0 = 176$ ft per sec.

b. Two seconds into the fall, the velocity is 54.75 feet per second. Find an exponential formula for D.

$v(2) = 54.75$

$D(2) = T(2) - v(2)$

$= 176 - 54.75 = 121.25$

t	D
0	176
2	121.25

$D(t) = 176 \times .83^t$

c. Find a formula for v.

$D = T - v$

$176 \times .83^t = 176 - v$

$-176 \qquad -176$

$(-1)(176 \times .83^t - 176) = -v\,(-1)$

$v(t) = 176 - 176 \times .83^t$

d. Express using functional notation the velocity 4 seconds into the fall, and then calculate that value.

$v(4) = 176 - 176 \times .83^4 = 92.47$ ft per sec.

Composition of Functions

Example 3: If $f(x) = 3x + 1$ and $g(x) = \frac{2}{x}$, find $f(g(2))$, $g(f(2))$, $f(g(x))$, and $g(f(x))$.

$f(g(2))$: $2 \rightarrow g(x) \rightarrow f(x)$

$f(g(x)) \neq g(f(x))$

$g(2) = \frac{2}{2} = 1$ $\quad f(1) = 3(1) + 1 = 4$ $\quad f(g(2)) = 4$

$g(f(2))$: $2 \rightarrow f(x) \rightarrow g(x)$

$f(2) = 3(2) + 1 = 7$ $\quad g(7) = \frac{2}{7}$ $\quad g(f(2)) = 2/7$

$f(x) = 3x + 1$

$g(x) = \boxed{\frac{2}{x}}$

$f(g(x)) = 3\left(\frac{2}{x}\right) + 1$

$= \boxed{\frac{6}{x} + 1}$

$f(x) = \boxed{3x + 1}$

$g(x) = \frac{2}{x}$

$\boxed{g(f(x)) = \frac{2}{3x+1}}$

Note that for most functions $f(g(x)) \neq g(f(x))$.

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Example 4: It was found that haddock grow to a maximum length of about 21 inches. Furthermore, the difference $D = D(t)$ between maximum length and length $L = L(t)$ at age t is an exponential function.

a. If initially a haddock is 4 inches long, find the initial value of D.

$D(t) = \text{Max length} - L(t)$

$\rightarrow D(t) = 21 - L(t)$

$L(0) = 4$

$D(0) = 21 - L(0)$
$= 21 - 4 = \boxed{17 \text{ inches}}$

b. It was found that a 6-year old haddock is about 15.8 inches long. Find a formula that gives L in terms of t.

$L(6) = 15.8$

$D(6) = 21 - L(6)$
$= 21 - 15.8$
$= 5.2$

t	$D(t)$
0	17
6	5.2

$D(t) = 17(.82)^t$

$D(t) = 21 - L(t)$

$17(.82)^t = 21 - L(t)$

$17(.82)^t - 21 = -L(t)$

$\boxed{L(t) = 21 - 17(.82)^t}$

c. The weight W in pounds of a haddock can be calculated from its length L using $W = 0.000293L^3$. Use function composition to find a formula for weight W as a function of age t.

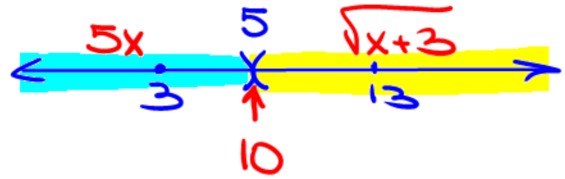
$W(L) = .000293L^3$

$W(t) = .000293(21 - 17(.82)^t)^3$

Piecewise-Defined Functions

A function that is defined by two (or more) equations over a specified domain is called a **piecewise function**.

Example 5: Suppose $f(x) = \begin{cases} \sqrt{x+3}, & x > 5 \\ 10, & x = 5 \\ 5x, & x < 5 \end{cases}$



a. Find $f(13)$.

$= \sqrt{13+3} = \sqrt{16}$

$= 4$

b. Find $f(3)$.

$= 5(3) = 15$

c. Find $f(5)$. $= 10$

Example 6: Here are two different models for U.S. public high school enrollments. One model is $N = 0.021t^2 - 0.09t + 11.74$, where N is enrollment in millions of students, t is time in years since 1987, and the formula is valid from 1987 to 1999. Another model is $N = -0.033t^2 + 0.46t + 13.37$, where N is enrollment in millions of students, t is time in years since 2000, and the formula is valid from 2000 to 2010.

a. Determine if these two functions give contradictory information.

NO

b. Find a way to write these formulas in terms of the same variable.

Year	2000	2001	2002	2003
t years since 2000	0	1	2	3
T years since 1987	13	14	15	16

$T = t + 13$

$t = T - 13$

$$N = \begin{cases} .021T^2 - .09T + 11.74 & 0 \le T \le 12 \\ -.033(T-13)^2 + .46(T-13) + 13.37 & 13 \le T \le 23 \end{cases}$$

c. Write N as a single piecewise-defined function.