Math 1311
Section 6.2
Rates of Change for Other Functions

- Notation $\frac{df}{dx}$ means the rate of change in f with respect to x.
- $\frac{df}{dx}$ tells how much f is expected to change if x is increased by 1 unit.
- $\frac{df}{dx}$ is a function of x.

Example 1: If $S = S(t)$ gives directed distance for an object as a function of time t, then

$\frac{dS}{dt}$ is the rate of change in directed distance with respect to time. This is velocity.

$\frac{dS}{dt}$ tells the additional distance we expect to travel in 1 unit of time.

We are currently located 100 miles south of Dallas, Texas, and traveling south with a velocity of 50 miles per hour, then in 1 additional hour we would expect to be 150 miles south of Dallas.

Example 2: If $V = V(t)$ is the velocity of an object as a function of time t, then

$\frac{dV}{dt}$ is the rate of change in velocity with respect to time. This is accelaration.

$\frac{dV}{dt}$ tells the additional velocity we expect to attain in 1 unit of time.

We are traveling with a velocity 50 miles per hour, and if we start to pass a track our acceleration might be 2 miles per hour each second. Then 1 second in the future, we would expect our velocity to be 52 miles per hour.

Example 3: If $T = T(D)$ denote the amount of income tax, in dollars, that you pay on an income of D dollars, then $\frac{dT}{dD}$ is the rate of change in tax with respect to the money you earn. This is marginal tax rate.

$\frac{dT}{dD}$ tells us the additional tax you expect to pay if you earn 1 additional dollar.

Suppose we have a tax liability of \$3000 and our marginal tax rate is 0.2. $1 \times .2 = .2$

If we earn an additional \$1, we would expect our tax liability to increase by 20c to a total of \$3000.2

$100 \times .2 = 20$

If we earn an additional \$100, we would expect our tax liability to increase by \$20 to a total of \$3020

Example 4: If $P = P(i)$ is the profit, in dollars, that you expect to earn on an investment of i dollars, and then $\frac{dP}{di}$ is the rate of change in profit with respect to dollars invested.

$\frac{dP}{di}$ tells how much additional profit to be expected if 1 additional dollar is invested.

This is marginal profit.

If your current investment in a project gives a profit of \$1000, and our marginal profit is 0.2, then we would expect that an additional investment of \$100 dollars would give additional profit of \$20 for a total profit of \$1020.

Fundamental Properties of Rates of Change

For a function $f = f(x)$ we will use the notation $\frac{df}{dx}$ to denote the rate of change in f with respect to x.

1. The expression $\frac{df}{dx}$ tells us how f changes in relation to x. It gives the additional value that is expected to be added to f if x increases by 1 unit.
2. When f is increasing, $\frac{df}{dx}$ is positive.
3. When f is decreasing, $\frac{df}{dx}$ is negative.
4. When f is not changing, $\frac{df}{dx}$ is zero.
5. When $\frac{df}{dx}$ is constant, then f is a linear function with slope $\frac{df}{dx}$.

$f(x) = mx + b$

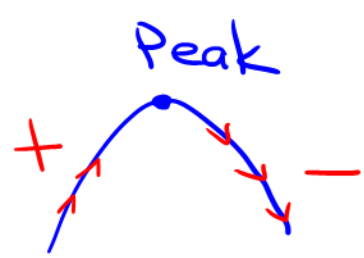
↑ slope

Example 5: What is the common term for the rate of change of each of the following phenomena?

a) Directed distance as a function of time velocity
b) Tax due as a function of income marginal tax rate
c) Profit as a function of dollars invested marginal profit
d) Velocity as a function of time acceleration

Exemple 6: Suppose $f = f(x)$. What is the sign of $\frac{df}{dx}$ in each of the following situations?

a) The function f is increasing. +
b) The function f has reached a peak. 0
c) The function f is decreasing. −
d) The graph of f is a horizontal line. 0



Example 7: Describe f if $\frac{df}{dx} = 10$.

f is increasing

f is a linear function with the slope $= 10$

$f(x) = 10x + b$

Example 8: What can be said about the graph of f if the graph of $\frac{df}{dx}$ is below the horizontal axis?

$\frac{df}{dx}$ is negative $\Rightarrow$ f is decreasing

Example 9: Let $s(a)$ denote sales generated by spending a dollars on advertising. My goal is to increase sales. If $\frac{ds}{da}$ is negative, should I spend more or less money on advertising?

Example 10: The price P of gasoline decreases to a minimum and starts to increase. What is the rate of change $\frac{dP}{dt}$ of the price with respect to time t at the time when the price reaches a minimum?