**Math 1311**
**Section 6.3**
**Estimating Rates of Change**

### Estimating Rates of Change in Tabulated Data

If $f = f(x)$ is a function given by a table of values, then $\frac{df}{dx}$, the rate of change of $f$ with respect to $x$, cannot without further information be calculated exactly. But it can be estimated using

$$\boxed{\text{Average rate of change in } f \text{ with respect to } x = \frac{\text{Change in } f}{\text{Change in } x}}$$

**Example 1:** The following table shows the location $S$(measured as distance in miles east of Los Angeles) of an airplane flying toward Denver. .5h

| | | |
|---|---|---|
| **Time** | 1:00 P.M. | 1:30 P.M. |
| **Distance from L.A.** | 360 miles | 612 miles |

Assuming the airplane is traveling at the same speed over the entire 30-minute time interval, find the velocity at 1:00 P.M.

$$\frac{dD}{dt} = \frac{612 - 360}{.5} = 504 \text{ miles per hour}$$

**Example 2:** The following table shows the population of reindeer on an island as of the given year.

| t | 0 | 5 | 10 | 1957 | 15 |
|---|---|---|---|---|---|
| **Date** | 1945 | 1950 | 1955 | | 1960 |
| **Population** | 40 | 165 | 678 | +2 | 2793 |

We let $t$ be the number of years since 1945, so that $t = 0$ corresponds to 1945, and we let $N = N(t)$ denote the population size.

a) Approximate $\frac{dN}{dt}$ for 1955 using the average rate of change from 1955 to 1960, and explain what this number means in practical terms.

$$\frac{dN}{dt} = \frac{2793 - 678}{15 - 10} = \frac{2115}{5} = 423 \text{ deer per year}$$

b) Use your work from part a) to estimate the population in 1957.

$678 + 2(423) = 1524$ deer in 1957

# of deer in 1955 2 years of change

**Rates of Change for Functions Given by Formulas**

**Example 3:** If air resistance is ignored, elementary physics can be used to show that the rock falls $S = 16t^2$ feet during $t$ second of fall.

a) Compute the velocity of the rock 2.5 seconds into the fall.
   To get more reliable answer keep the interval short!
b) Repeat the calculation using the automated features provided by the calculator.

| | | |
|---|---|---|
| **t** | 2.5 | 2.50001 |
| **S** | 100 | 100.0008 |

$S(2.5) = 16(2.5)^2 = 100$

$S(2.50001) = 16(2.50001)^2 = 100.0008$

$\frac{dS}{dt} = \frac{100.0008 - 100}{2.50001 - 2.5} = 80$ ft per sec.

**Example 4:** If $f$ is the linear function $f = 7x - 3$, what is the value of $\frac{df}{dx}$?

ROC = slope = 7

**Example 5:** Suppose $f = f(x)$ satisfies $f(2) = 5$ and $f(2.005) = 5.012$. Estimate the value of $\frac{df}{dx}$ at $x = 2$.

| x | 2 | 2.005 |
|---|---|---|
| f(x) | 5 | 5.012 |

$\frac{df}{dx} = \frac{(5.012 - 5)}{(2.005 - 2)} = \boxed{2.4}$

**Example 6:** By direct calculation, estimate the value of $\frac{df}{dx}$ for $f(x) = x^2 + 1$ at $x = 3$. Use an increment of 0.0001.

+.0001

| x | 3 | 3.0001 |
|---|---|---|
| f(x) | 10 | 10.00060001 |

$f(3) = (3)^2 + 1 = 10$

$f(3.0001) = (3.0001)^2 + 1$

$=$

$$\frac{df}{dx} = \frac{10.00060001 - 10}{.0001} = 6.0001$$

**Example 7:** Make a graph of $x^3 - x^2$ and use the calculator to estimate its rate of change at $x = 3$. The recommended horizontal span is 0 to 4 and vertical span is 0 to 30.

$\frac{df}{dx}$ for $x = 3$ $= 21$

xmin Xmax Ymin Ymax

**Example 8:** Make a graph of $x + \frac{1}{x}$ and use the calculator to estimate its rate of change at $x = 3$. The recommended horizontal span is 1 to 4 and vertical span is 0 to 5.

$$\frac{df}{dx} = .\overline{8} \approx .8889$$

**Example 9:** Make a graph of $3^{-x}$ and use the calculator to estimate its rate of change at $x = 3$. The recommended horizontal span is 0 to 4 and vertical span is -1 to 1.

$$\frac{df}{dx} = -.0407$$