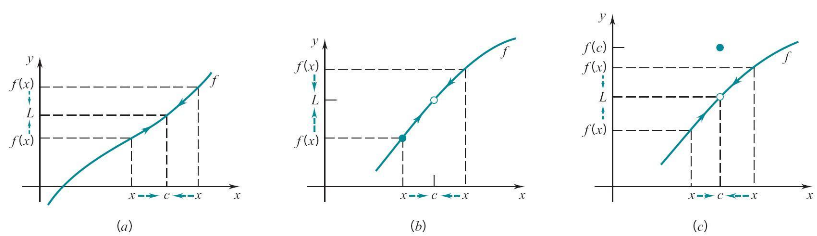
Lecture 1 Section 2.1 The Ideal of Limit Section 2.2 Definition of Limit

Jiwen He

1 Section 2.1 The Ideal of Limit

1.1 The Ideal of Limit

Graphical Introduction to Limit

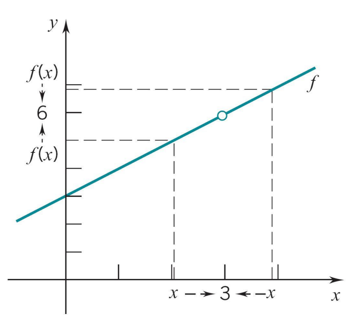


$$\lim_{x \to c} f(x) = L$$

- In taking the limit of a function f as x approaches c, it does not matter whether f is defined at c and, if so, how it is defined there.
- The only thing that matters is the values that f takes on when x is near c.

1.2 Examples

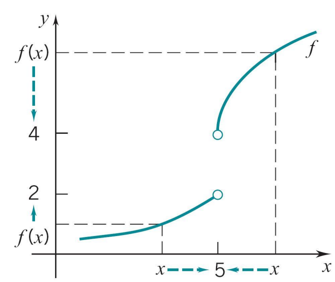
Example: $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$



Let $f(x) = \frac{x^2-9}{x-3}$. Use the graph of f to find

(a) $f(3)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$ (d) $\lim_{x\to 3} f(x)$

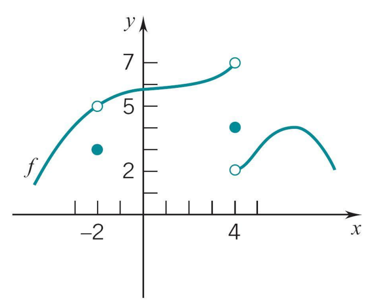
Example



Use the graph of f to find

(a) $f(5)$ (b) $\lim_{x\to 5^-} f(x)$ (c) $\lim_{x\to 5^+} f(x)$ (d) $\lim_{x\to 5} f(x)$

Example

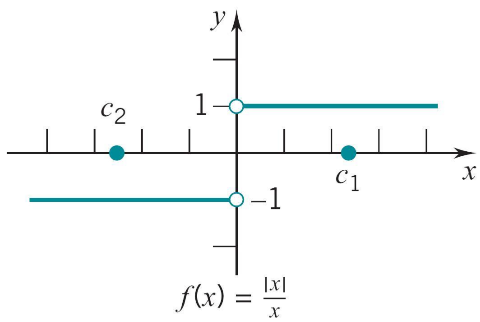


Use the graph of f to find

(a) $f(-2)$ (b) $\lim_{x\to -2^-} f(x)$ (c) $\lim_{x\to -2^+} f(x)$ (d) $\lim_{x\to -2} f(x)$

(e) $f(4)$ (f) $\lim_{x\to 4^-} f(x)$ (g) $\lim_{x\to 4^+} f(x)$ (h) $\lim_{x\to 4} f(x)$

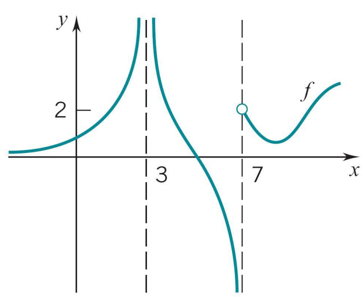
Example: $\lim_{x\to 0} \frac{|x|}{x}$



Let $f(x) = \frac{|x|}{x}$. Use the graph of f to find

(a) $f(0)$ (b) $\lim_{x\to 0^-} f(x)$ (c) $\lim_{x\to 0^+} f(x)$ (d) $\lim_{x\to 0} f(x)$

Example



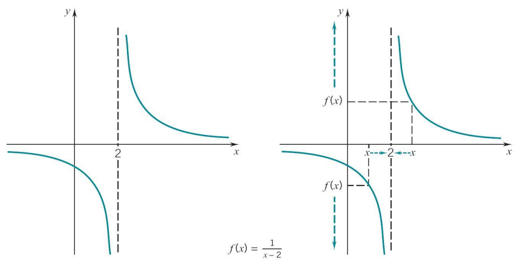
Use the graph of f to find

(a) $f(3)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$ (d) $\lim_{x\to 3} f(x)$

(e) $f(7)$ (f) $\lim_{x\to 7^-} f(x)$ (g) $\lim_{x\to 7^+} f(x)$ (h) $\lim_{x\to 7} f(x)$

Example: $\lim_{x\to 2} \frac{1}{x-2}$

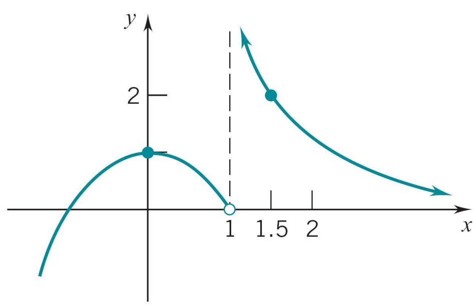
x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$	−2	−10	−100	−1000		1000	100	10	2



Let $f(x) = \frac{1}{x-2}$. Use the graph of f to find

(a) $f(2)$ (b) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 2^+} f(x)$ (d) $\lim_{x\to 2} f(x)$

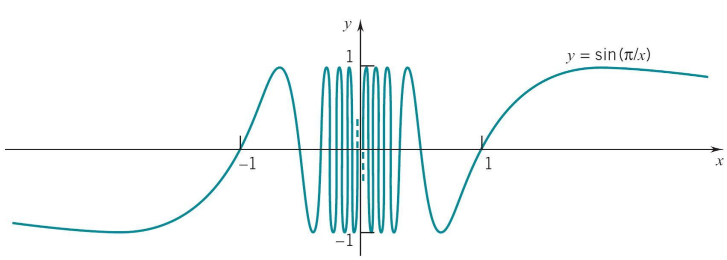
Example



Let $f(x) = \begin{cases} 1-x^2 & x<1 \\ \frac{1}{x-1} & x>1 \end{cases}$. Use the graph of f to find

(a) $f(1)$ (b) $\lim_{x\to 1^-} f(x)$ (c) $\lim_{x\to 1^+} f(x)$ (d) $\lim_{x\to 1} f(x)$

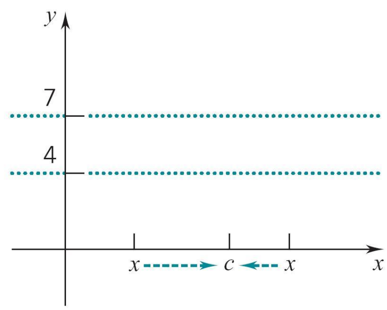
Example: $\lim_{x\to 0} \sin\frac{\pi}{x}$



Let $f(x) = \sin \frac{\pi}{x}$. Use the graph of f to find

(a) $f(0)$ (b) $\lim_{x \to 0^-} f(x)$ (c) $\lim_{x \to 0^+} f(x)$ (d) $\lim_{x \to 0} f(x)$

Example



Let $f(x) = \begin{cases} 7, & x \text{ rational} \\ 4, & x \text{ irrational} \end{cases}$. Use the graph of f to find

(a) $f(8)$ (b) $\lim_{x \to 8^-} f(x)$ (c) $\lim_{x \to 8^+} f(x)$ (d) $\lim_{x \to 8} f(x)$

1.3 Theorem

An Important Theorem

Theorem 1.

$$\lim_{x \to c} f(x) = L$$

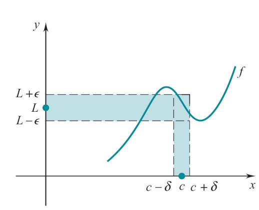
if and only if both

$$\lim_{x \to c^-} f(x) = L \quad \textit{and} \quad \lim_{x \to c^+} f(x) = L$$

2 Section 2.2 Definition of Limit

2.1 Definition of Limit

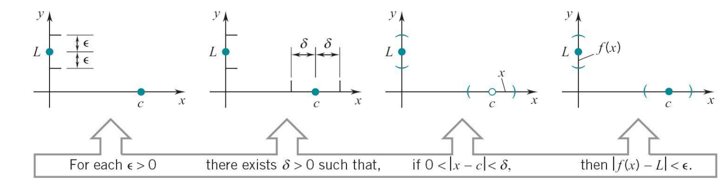
Definition of Limit: ϵ, δ statement



We say that

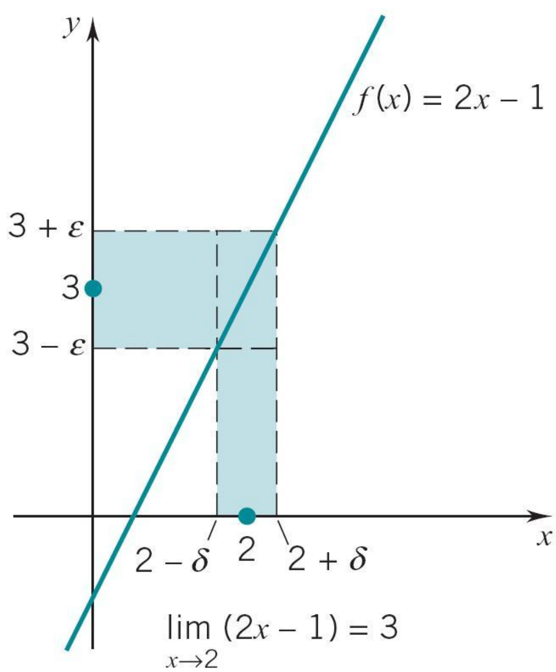
$$\lim_{x \to c} f(x) = L$$

if



2.2 Examples

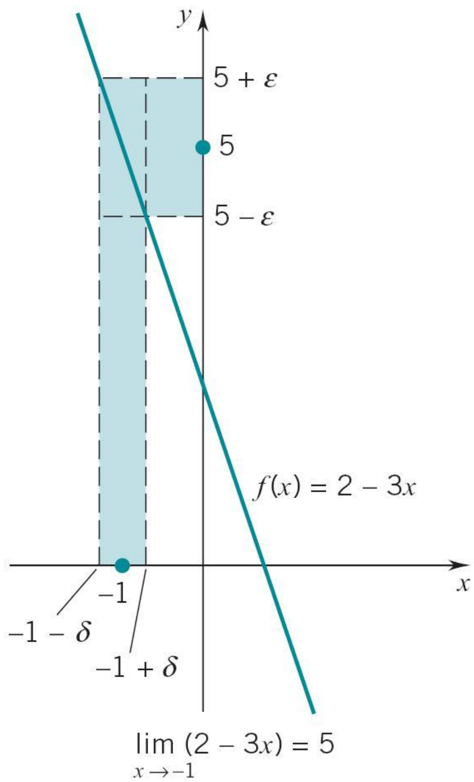
Example: $\lim_{x \to 2} (2x - 1) = 3$
Show that $\lim_{x \to 2} (2x - 1) = 3$.



Let $\epsilon > 0$. We choose $\delta = \frac{1}{2}\epsilon$ such that

$$\text{if } \; 0 < |x - 2| < \delta, \qquad \text{then } \; |(2x - 1) - 3| < \epsilon$$

Example: $\lim_{x \to -1} (2 - 3x) = 5$
Show that $\lim_{x \to -1} (2 - 3x) = 5$.

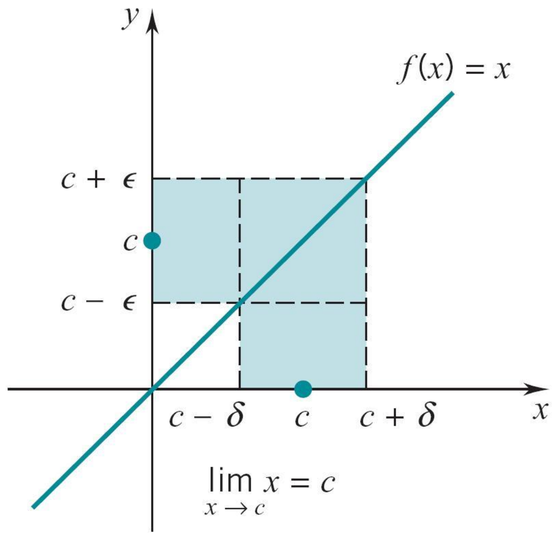


Let $\epsilon > 0$. We choose $\delta = \frac{1}{3}\epsilon$ such that

$$\text{if} \;\; 0 < |x - (-1)| < \delta, \qquad \text{then} \;\; |(2 - 3x) - 5| < \epsilon$$

2.3 Three Basic Limits

$\lim x = c$
Show that $\lim_{x \to c} x = c$.

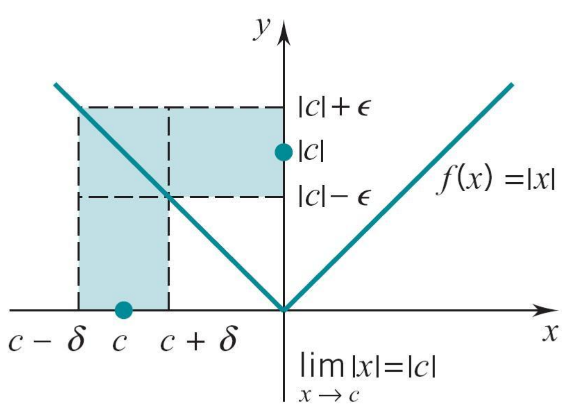


Let $\epsilon > 0$. We choose $\delta = \epsilon$ such that

$$\text{if } \ 0 < |x - c| < \delta, \qquad \text{then } \ |x - c| < \epsilon$$

$\lim_{x\to c} |x| = |c|$

Show that $\lim_{x\to c} |x| = |c|$.

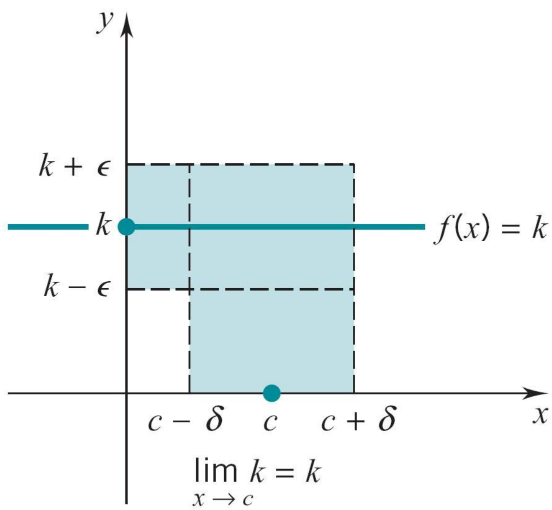


Let $\epsilon > 0$. We choose $\delta = \epsilon$ such that

$$\text{if } \ 0 < |x - c| < \delta, \qquad \text{then } \ ||x| - |c|| < \epsilon$$

$\lim_{x \to c} k = k$

Show that $\lim_{x \to c} k = k$.



Let $\epsilon > 0$. We can choose any number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - c| < \delta, \qquad \text{then} \quad |k - k| < \epsilon$$