Lecture 5

Section 3.1 The Derivative

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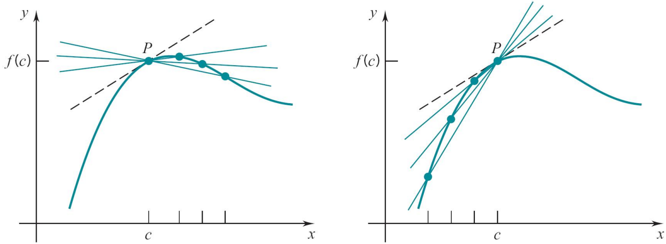


Homework and Quizzes

Homework 2 & 3

- Homework 2 is due today in lab.
- Homework 3 is due September 16th in lab.

Online Quizzes

- Quizzes 1 and 2 have expired!
- Quiz 3 is posted and due on this Friday before 11:59 PM!
- Quiz 4 is posted and due 9/19!

Weekly Written Quizzes in Lab

- Quizzes will be given every week on Thursday in lab beginning THIS WEEK.
- The weekly written quizzes form is posted on the course homepage. You must print out this form and BRING it to class every Thursday.

Daily Grades

- Daily grades start Today.
- The daily grades form is posted on the course homepage. You must print out this form and BRING it to class every day.

Quiz 1

Quiz 1

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$$

a. 1

b. 2

c. -1

d. None of these

Quiz 2

Quiz 2

Classify the discontinuity at $x = 1$ for

$$f(x) = \frac{x^2 - 1}{x - 1}.$$

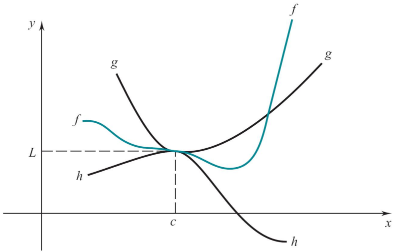
a. Jump

b. Infinite

c. Removable

d. None of these

The Pinching Theorem



Theorem

Let $p > 0$. Suppose that, for all x such that $0 < |x - c| < p$

$$h(x) \leq f(x) \leq g(x).$$

If

$$\lim_{x \to c} h(x) = L \quad \textit{and} \quad \lim_{x \to c} g(x) = L.$$

then

$$\lim_{x \to c} f(x) = L.$$

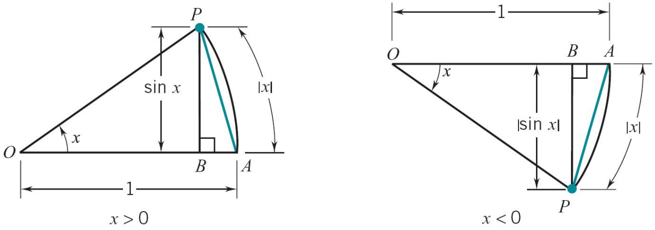
The Pinching Theorem: Continuity of Sine and Cosine

Theorem

$$\lim_{x\to 0} \sin x = 0, \quad \lim_{x\to 0} \cos x = 1,$$

$$\lim_{x\to c} \sin x = \sin c, \quad \lim_{x\to c} \cos x = \cos c.$$

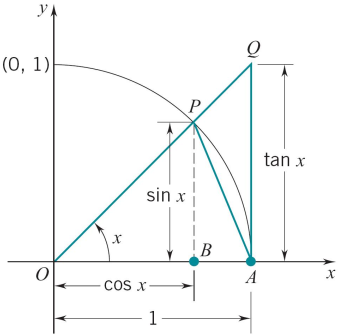
$$0 < |\sin x| < |x|.$$



The Pinching Theorem: Trigonometric Limits

Theorem

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0.$$



Proof.

Use Geometric argument to get

$$\cos x < \frac{\sin x}{x} < 1,$$

then apply the pinching theorem.

More Trigonometric Limits

Theorem

$$\lim_{x\to 0} \frac{x}{\sin x} = 1 \quad \Rightarrow \quad \sin x \approx x \textit{ for } x \textit{ near } 0.$$

Theorem

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \quad \Rightarrow \quad \cos x \approx 1 - \frac{1}{2}x^2 \textit{ for } x \textit{ near } 0.$$

Theorem

For any number $\alpha \neq 0$,

$$\lim_{x\to 0} \frac{\sin \alpha x}{\alpha x} = 1, \quad \lim_{x\to 0} \frac{1-\cos \alpha x}{\alpha x} = 0,$$
$$\lim_{x\to 0} \frac{\alpha x}{\sin \alpha x} = 1, \quad \lim_{x\to 0} \frac{1-\cos \alpha x}{(\alpha x)^2} = \frac{1}{2}.$$

Quiz 3

Quiz 3

What day is today?

a. Monday

b. Wednesday

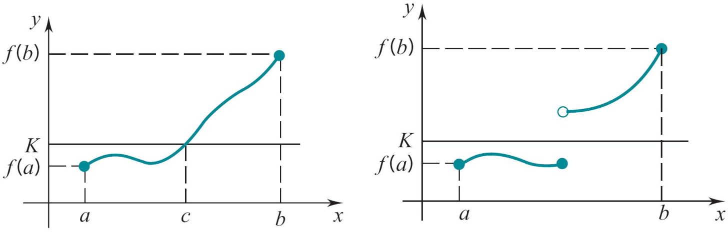
c. Friday

d. None of these

The Intermediate-Value Theorem

Theorem

If f is continuous on $[a, b]$ and K is any number between $f(a)$ and $f(b)$, then there is at least one number c in the interval (a, b) such that $f(c) = K$.



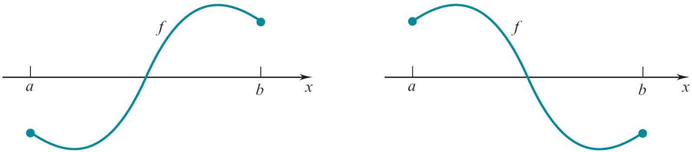
The Intermediate-Value Theorem: Roots of Equation

Theorem

If f is continuous on $[a, b]$ and

$$f(a) < 0 < f(b), \quad \text{or} \quad f(b) < 0 < f(a),$$

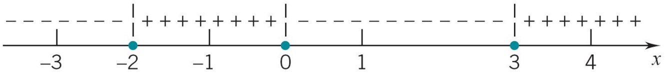
then the equation $f(x) = 0$ has at least a root in (a, b).



The Intermediate-Value Theorem: Solution of Inequality

Solve the inequality

$$x^3 - x^2 - 6x > 0$$

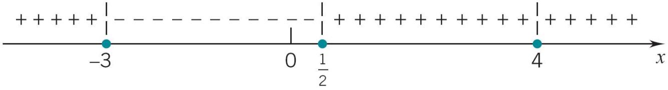


Solution: $(-2, 0) \cup (3, \infty)$.

Solution of Inequality

Solve the inequality

$$(x+3)^3(2x-1)(x-4)^2 \leq 0$$

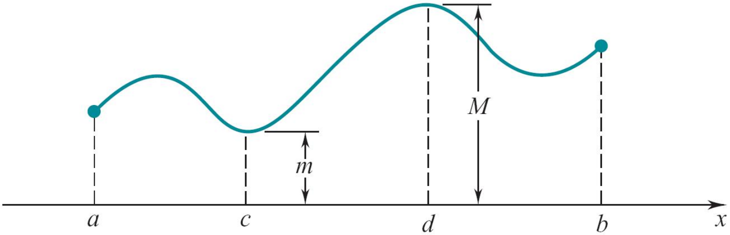


Solution: $[-3, 1/2]$.

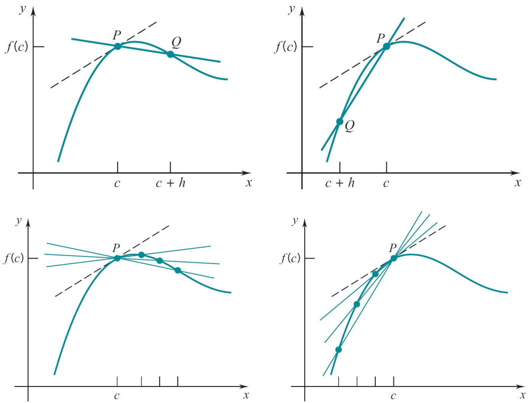
The Extreme-Value Theorem

Theorem

A function f continuous on a bounded closed $[a, b]$ takes on both a maximum value M and a minimum value m.



Secant Lines vs. Tangent Lines

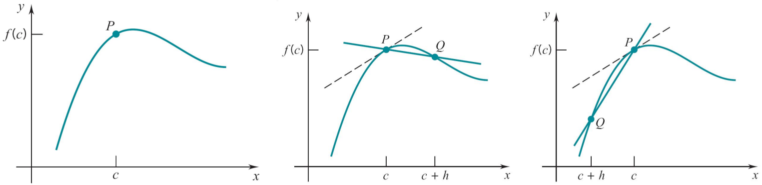


Definition

The slope of the graph at the point $(c, f(c))$ is given by

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}, \qquad \text{provided the limit exists}$$

Derivative and Differentiation



Definition

A function f is *differentiable* at c if

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \quad \text{exists.}$$

If this limit exists, it is called the *derivative* of f at c, and is denoted by $f'(c)$.

Derivative as Function

Definition

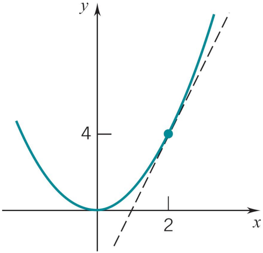
The *derivative* of a function f is the function f' with value at x given by:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}, \quad \text{provided the limit exists.}$$

To *differentiate* a function f is to find its derivative.

Examples

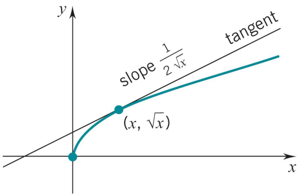
$$f(x) = x^2, \quad f'(x) = 2x.$$



Derivative as Function: More Examples

Examples

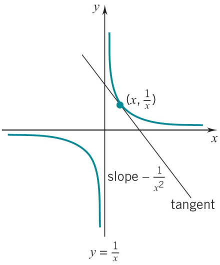
$$f(x) = \sqrt{x}, \quad f'(x) = \frac{1}{2\sqrt{x}}.$$



square root function

Examples

$$f(x) = \frac{1}{x}, \quad f'(x) = -\frac{1}{x^2}.$$

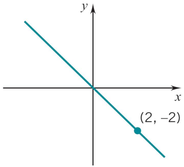


Line Functions

For a line function $f(x) = mx + b$, the derivative $f'(x) = m$.

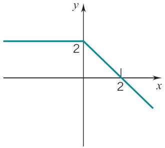
Examples

$$f(x) = -x, \quad f'(x) = -1.$$



Examples

$$f(x) = \begin{cases} 2, & x \le 0, \\ -x + 2, & x > 0. \end{cases} \quad f'(x) = \begin{cases} 0, & x < 0, \\ -1, & x > 0. \end{cases}$$



Nondifferentiability: Discontinuity

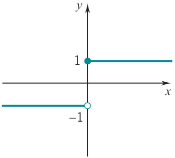
A function f is *not differentiable* at c if

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \quad \text{does not exist.}$$

Jump discontinuity at 0

as $x \to 0^-$, $f'(x) \to \infty$;

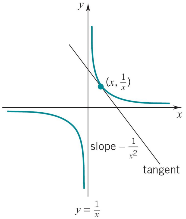
as $x \to 0^+$, $f'(x) \to 0$;



Infinite discontinuity at 0

as $x \to 0^-$, $f'(x) \to \infty$;

as $x \to 0^+$, $f'(x) \to \infty$;



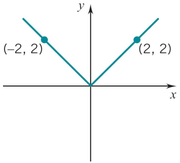
Nondifferentiability: Corner Points

A function f is *not differentiable* at c if

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \quad \text{does not exist.}$$

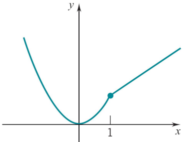
Corner point at 0

as $x \to 0^-$, $f'(x) \to -1$;
as $x \to 0^+$, $f'(x) \to 1$;



Corner point at 1

as $x \to 1^-$, $f'(x) \to 2$;
as $x \to 1^+$, $f'(x) \to \frac{1}{2}$;



no derivative at 1

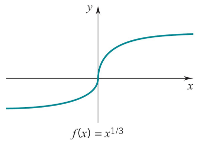
Nondifferentiability: Vertical tangents

A function f is *not differentiable* at c if

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \quad \text{does not exist.}$$

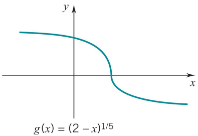
vertical tangent at 0

as $x \to 0$, $f'(x) \to \infty$.



vertical tangent at 2

as $x \to 2$, $f'(x) \to -\infty$.



Nondifferentiability: Vertical Cusps

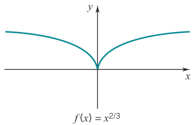
A function f is *not differentiable* at c if

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \quad \text{does not exist.}$$

vertical cusp at 0

as $x \to 0^-$, $f'(x) \to -\infty$;

as $x \to 0^+$, $f'(x) \to \infty$.



vertical cusp at 1

as $x \to 1^-$, $f'(x) \to \infty$;

as $x \to 1^+$, $f'(x) \to +\infty$.