Lecture 9

Section 3.4 Derivative as a Rate of Change
Section 3.8 Rates of Change per Unit Time

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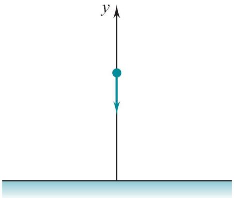


$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

Test 1

- Test 1 - updated due to ike.
- October 7-9 in CASA
- Loggin to CourseWare to reserve your time to take the exam.

Online Quizzes 3 and 4

- Online Quizzes 3 and 4 are available on CourseWare.
- The due dates for Quizzes have been extended.

Homework 4

- Homework 3 is cancelled due to Ike.
- Homework 4 is due September 30th in lab.

Homework Help Session

- Homework Help Session by Prof. Morgan
- Tonight 8:00 - 10:00pm in 100 SEC

Review for Test 1

- Review for Test 1 by the Scholars Community.
- Friday, September 26 2:30–3:30 in the Basement of the C-site.

Quiz 1

Quiz 1

Let $F(x) = f(g(x))$, $g(2) = -3$, $g'(2) = 5$, $f'(-3) = -1$. Find $F'(2)$.

a. 5

b. 3

c. -3

d. -5

e. None of these

Quiz 2

Quiz 2

Let $x^2 + (x+y)^4 = 5$. Give $\frac{dy}{dx}$ when at the point $(2, -1)$.

a. 2

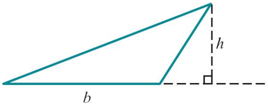
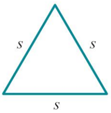
b. 1

c. -2

d. -1

e. None of these

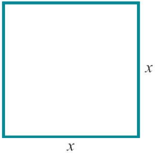
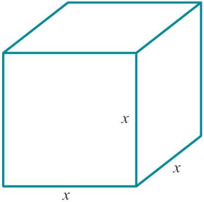
Triangle

Triangle	Equilateral Triangle
	
area $= \frac{1}{2}bh$	area $= \frac{1}{4}\sqrt{3}\,s^2$

The rate of change of the area A of an equilateral triangle with respect to its side length s

$$\frac{dA}{ds} = \frac{1}{2}\sqrt{3}s.$$

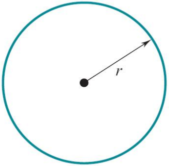
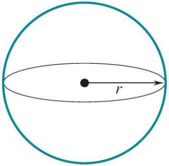
Square

Square	Cube
	
area $= x^2$ perimeter $= 4x$ diagonal $= x\sqrt{2}$	volume $= x^3$ surface area $= 6x^2$

The rate of change of the volume V of a cube with respect to its side length x

$$\frac{dV}{dx} = 3x^2.$$

Circle

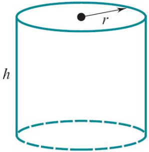
Circle	Sphere
	
area $= \pi r^2$ circumference $= 2\pi r$	volume $= \frac{4}{3}\pi r^3$ surface area $= 4\pi r^2$

The rate of change of the volume V of a sphere with respect to its radius r

$$\frac{dV}{dr} = 4\pi r^2.$$

Right Circular Cylinder

Right Circular Cylinder

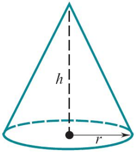


volume $= \pi r^2 h$

lateral area $= 2\pi r h$

total surface area $= 2\pi r^2 + 2\pi r h$

Right Circular Cone



volume $= \frac{1}{3}\pi r^2 h$

slant height $= \sqrt{r^2 + h^2}$

lateral area $= \pi r \sqrt{r^2 + h^2}$

total surface area $= \pi r^2 + \pi r \sqrt{r^2 + h^2}$

Suppose that the height h of a right circular cone is always twice the radius r. The rate of change of the volume V of this cone with respect to h

$$h = 2r, \quad V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3, \quad \frac{dV}{dh} = \frac{1}{4}\pi h^2.$$

Velocity and Acceleration

- Let $y(t)$ be the position of a moving object at time t.
- The rate of change of position y with respect to time t is called the velocity,
$$v(t) = y'(t)$$
- The rate of change of velocity v with respect to time t is called the acceleration,
$$a(t) = v'(t) = y''(t)$$

Free Fall

- Let $y(t)$ be the height of the falling object at time t.
- Galileo's formula

$$y(t) = -\frac{1}{2}gt^2 + v_0 t + y_0.$$

Since $y(0) = y_0$, the constant y_0 is the height at time 0.

- Differentiation gives the velocity,

$$v(t) = y'(t) = -gt + v_0.$$

Since $v(0) = v_0$, the constant v_0 is the velocity at time 0.

- A second differentiation gives the acceleration,

$$a(t) = v'(t) = y''(t) = -g.$$

The constant g is a gravitatinal constant.

Near the Surface of the Earth

- The gravitatinal constant g is 32 feet per second per second

$$y(t) = -16t^2 + v_0 t + y_0 \quad \text{(distance in feet).}$$

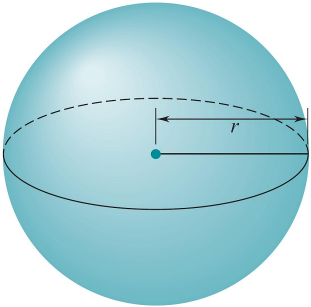
- The gravitatinal constant g is 9.8 meters per second per second

$$y(t) = -4.9t^2 + v_0 t + y_0 \quad \text{(distance in meters).}$$

Example

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. In how many seconds does it attain maximum height?

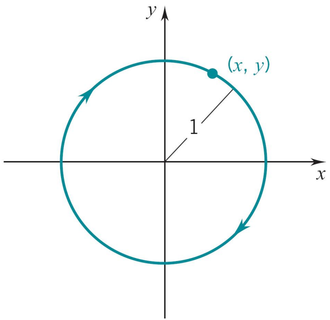
Expanding Balloon



Example

A spherical balloon is expanding. If the radius is increasing at the rate of 2 inches per minute, at what rate is the volume increasing when the radius is 5 inches?

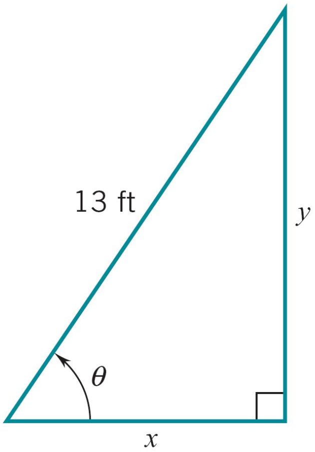
Circular Motion



Example

An object is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$. As it passes through the point $(1/2, \sqrt{3}/2)$, its y-coordinate is decreasing at the rate of 3 unit per second. At what rate is the x-coordinate changing at this point?

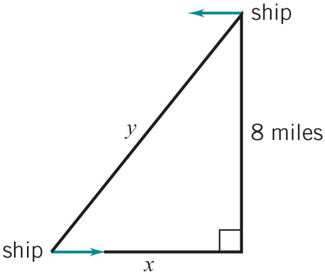
Ladder Pulled Away



Example

A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at the rate of 0.1 foot per second, how fast the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 feet above the ground?

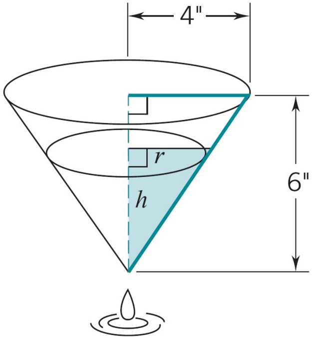
Ships Cruising



Example

Two ships, one heading west and the other east, approach each other on parallel courses 8 nautical miles apart. Given that each ship is cruising at 20 nautical miles per hour (knots), at what rate is the distance between them diminishing when they are 10 nautical miles apart?

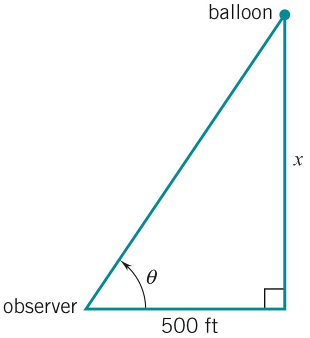
Leaking Cup



Example

A conical paper cup 8 inches across the top and 6 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep?

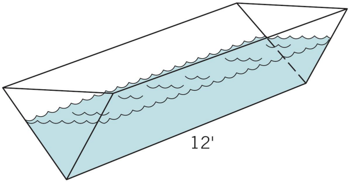
Flying Balloon



Example

A balloon leaves the ground 500 feet away from an observer and rises vertically at the rate of 140 feet per minute. At what rate is the angle of inclination of the observer's line of sight increasing at the instant when the balloon is exactly 500 feet above the ground?

Water Trough



Example

A water trough with vertical cross section in the shape of an equilateral triangle is being filled at a rate of 4 cubic feet per minute. Given that the trough is 12 feet long, how fast is the level of the water rising at the instant the water reaches a depth of 1.5 feet?