Lecture 10

Section 3.9 Differentials; Newton Approximation
Section 4.1 Mean-Value Theorem

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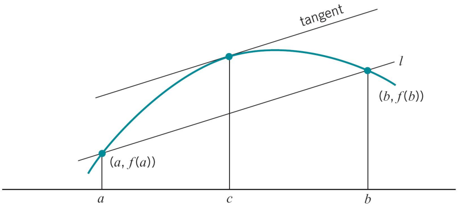


Test 1

- Test 1 - updated due to ike.
- October 7-9 in CASA
- Loggin to CourseWare to reserve your time to take the exam.

Online Quizzes

- Online Quizzes are available on CourseWare.
- The due dates for Quizzes have been extended.

Review for Test 1

- Review for Test 1 by by Prof. Morgan.
- Tonight 8:00 - 10:00pm in 100 SEC

Quiz 1

Quiz 1

$$\lim_{x \to 0} \frac{\sin(7x)}{\sin(5x)}$$

a. 1

b. $1/3$

c. $7/5$

d. 2

e. None of these

Quiz 2

Quiz 2

Where is $f(x) = \dfrac{x-1}{x^2-1}$ continuous?

a. everywhere

b. $x = 1, -1$

c. $x = 1$

d. everywhere except $x = -1$

e. None of these

Quiz 3

Quiz 3

Find the slope of the tangent line to the graph of $f(x) = x^2 + 3x$ at $x = 1$.

a. 4

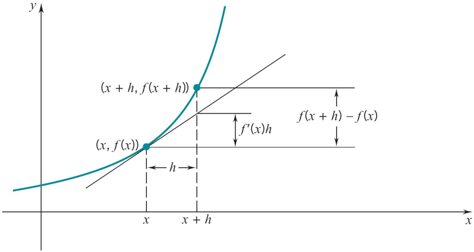
b. 5

c. 6

d. 7

e. None of these

Differentials

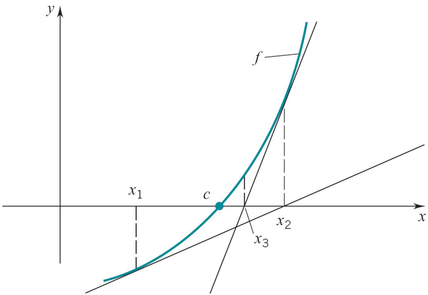


- increment: $\Delta f = f(x+h) - f(x)$
- differential: $df = f'(x)h$

$$\Delta f \approx df$$

in the sense that $\dfrac{\Delta f - df}{h}$ tends to 0 as $h \to 0$.

Newton-Raphson Approximation



Newton Method

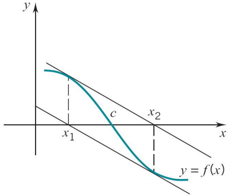
Let the number c be a solution (root) of an equation $f(x) = 0$. The Newton-Raphson method

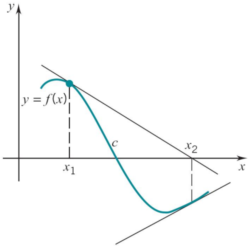
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

generates a sequence of approximations $x_1, x_2, \cdots, x_n, \cdots$ that will "converge" to the root c

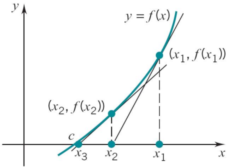
Convexity Conditions for the Convergence

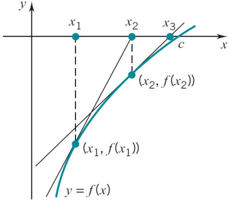
looping or divergent





$$f(x)f''(x) > 0$$





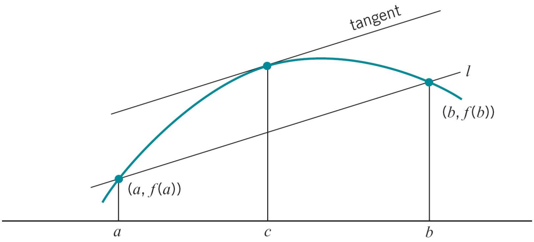
Example: Estimate $\sqrt{3}$

n	x_n	$x_{n+1} = \frac{x_n^2+3}{2x_n}$
1	2	1.75000
2	1.75000	1.73214
3	1.73214	1.73205

The number $\sqrt{3}$ is a root of the equation $x^2 - 3 = 0$. Estimate $\sqrt{3}$ by applying the Newton method to the function $f(x) = x^2 - 3$ starting at $x_1 = 2$:

$$x_{n+1} = x_n - \frac{x_n^2 - 3}{2x_n} = \frac{x_n^2 + 3}{2x_n}.$$

The Mean-Value Theorem



Theorem

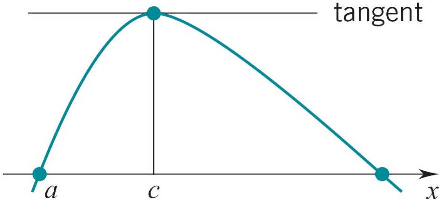
If f is differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$, then there is at least one number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a).$$

Rolle's Theorem



Theorem

Let f be differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$. If $f(a) = f(b) = 0$, then there is at least one number c in (a, b) at which

$$f'(c) = 0.$$