Lecture 11

Section 4.1 Mean-Value Theorem
Section 4.2 Increasing and Decreasing Functions

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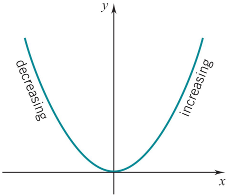


Test 1

- Test 1 - updated due to ike.
- October 7-9 in CASA

Quiz 1

Quiz 1

Use 1 iteration of Newton's method to approx. a solution to

$$x^3 - 4x + 1 = 0$$

from a guess of $x_0 = 2$.

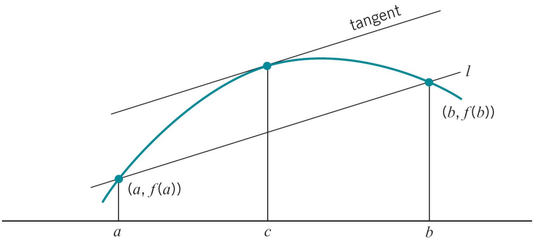
a. 15/8

b. 17/8

c. 33/16

d. None of these

The Mean-Value Theorem



Theorem

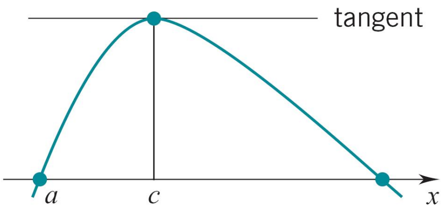
If f is differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$, then there is at least one number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a).$$

Rolle's Theorem

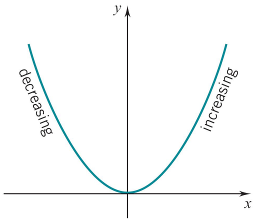


Theorem

Let f be differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$. If $f(a) = f(b) = 0$, then there is at least one number c in (a, b) at which

$$f'(c) = 0.$$

Increasing and Decreasing Functions



Definition

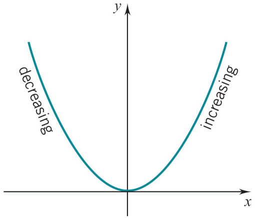
- A function f is increasing on an interval I if

$$f(a) < f(b), \quad \forall a, b \in I \text{ with } a < b.$$

- A function f is decreasing on an interval I if

$$f(a) > f(b), \quad \forall a, b \in I \text{ with } a < b.$$

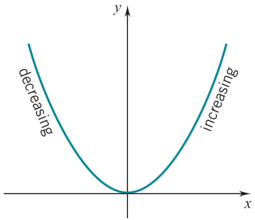
Sign of Derivative



Theorem

- *A function f is increasing on an interval I if*
  - *f is continuous and*
  - *$f'(x) > 0$ at all but finitely many values in I.*
- *A function f is decreasing on an interval I if*
  - *f is continuous and*
  - *$f'(x) < 0$ at all but finitely many values in I.*

Example

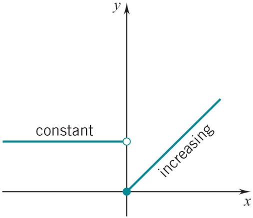


$$f(x) = x^2,$$

$$f'(x) = 2x \begin{cases} < 0 & x < 0 \\ > 0, & x > 0 \end{cases}$$

- f is continuous everywhere.
- f is decreasing on $(-\infty, 0]$, increasing on $[0, \infty)$.

Example

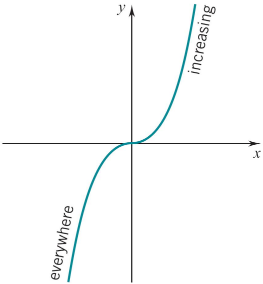


$$f(x) = \begin{cases} 1, & x < 0 \\ x, & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$$

- f has a discontinuity at $x = 0$.
- f is constant on $(-\infty, 0)$, increasing on $[0, \infty)$.

Example

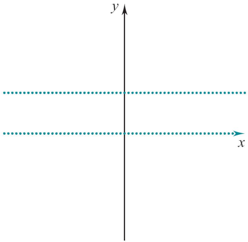


$$f(x) = x^3,$$

$$f'(x) = 3x^2 > 0$$

- f is continuous everywhere.
- f is everywhere increasing.

Example

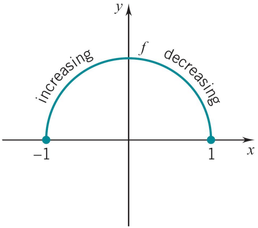


Dirichlet function

$$f(x) = \begin{cases} 1, & \text{x rational} \\ 0, & \text{x irrational} \end{cases}$$

- f is discontinuous everywhere.
- there is no interval where f is increasing or decreasing.

Example

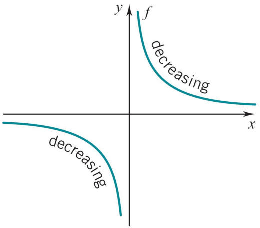


$$f(x) = \sqrt{1-x^2},$$

$$f'(x) = -\frac{x}{\sqrt{1-x^2}} \begin{cases} > 0 & -1 < x < 0 \\ < 0, & 0 < x < 1 \end{cases}$$

- f is continuous on [-1,1].
- f is increasing on $[-1, 0]$, decreasing on $[0, 1]$.

Example

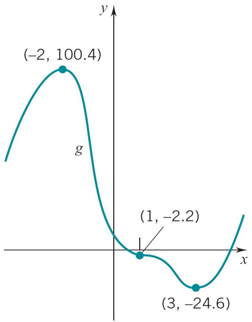


$$f(x) = \frac{1}{x},$$

$$f'(x) = -\frac{1}{x^2} < 0$$

- f is discontinuous at $x = 0$.
- f is decreasing on $(-\infty, 0)$ and on $(0, \infty)$.

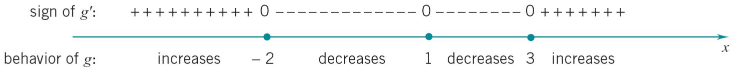
Example



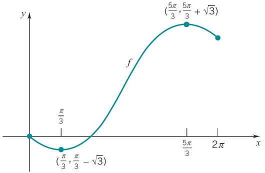
$$f(x) = \frac{4}{5}x^5 - 3x^4 - 4x^3 + 22x^2 - 24x + 6,$$

$$f'(x) = 4(x+2)(x-1)^2(x-3)$$

- f is continuous everywhere.
- f is increasing on $(-\infty, -2]$, decreasing on $[-2, 3]$, and increasing on $[3, \infty)$.



Example

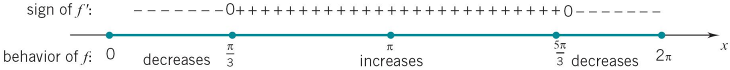


$$f(x) = x - 2\sin x, \quad 0 \le x \le 2\pi,$$

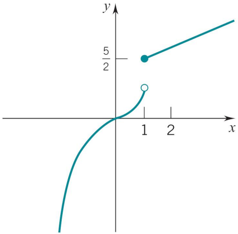
$$f'(x) = 1 - 2\cos x, \quad 0 \le x \le 2\pi.$$

$$f'(x) = 0 \text{ at } x = \pi/3, 5\pi/3.$$

- f is continuous on $[0, 2\pi]$.
- f is decreasing on $[0, \pi/3]$, increasing on $[\pi/3, 5\pi/3]$, and decreasing on $[5\pi/3, 2\pi]$.



Example

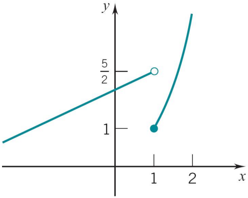


$$f(x) = \begin{cases} x^3, & x < 1 \\ \frac{1}{2}x + 2, & x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2, & x < 1 \\ \frac{1}{2}, & x > 1 \end{cases}$$

- f has a discontinuity at $x = 1$.
- f is increasing on $(-\infty, 1)$ and on $[1, \infty)$.
- Note that f is increasing on $(-\infty, \infty)$.

Example

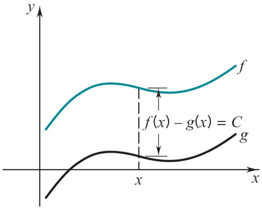


$$f(x) = \begin{cases} \frac{1}{2}x + 2, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{2}, & x < 1 \\ 3x^2, & x > 1 \end{cases}$$

- f has a discontinuity at $x = 1$.
- f is increasing on $(-\infty, 1)$ and on $[1, \infty)$.
- Note that f is NOT increasing on $(-\infty, \infty)$.

Equality of Derivatives



Theorem

$$f'(x) = g'(x), \quad \forall x \in I$$

if and only if

$$f(x) = g(x) + C, \quad \forall x \in I.$$

with C a constant .