## Lecture 13

## Section 4.5 Some Max-Min Problems

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- The written questions on Test 1 have been graded on Monday, and should appear as a separate column in your CourseWare gradebook by Wednesday.
- You will have to add the two columns to get your total score on the exam.


## Quiz 1

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Give the number of critical values of $f$.
a. 2
b. 3
c. 4
d. 5
e. None of these


## Quiz 2

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Give the number of local minima of $f$.
a. 1
b. 2
c. 3
d. 4
e. None of these


## Quiz 3

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Give the number of local maxima of $f$.
a. 1
b. 2
c. 3
d. 4
e. None of these


## Quiz 4

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Give the number of intervals of increase of $f$.
a. 1
b. 2
c. 3
d. 4
e. None of these


## Quiz 5

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Give the number of intervals of decrease of $f$.
a. 1
b. 2
c. 3
d. 4
e. None of these


## Quiz 6

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at -3 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 7

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at -2 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 8

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at -1 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 9

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at 0 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 10

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at 1 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 11

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at 2 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Quiz 12

Assume the domain of $f$ is all real numbers. The graph of $f^{\prime}(x)$ is shown below. Classify the critical value at 3 or state that the value is not a critical value.
a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these


## Example 1




Example 1 An isosceles triangle has a base of 6 units and a height of 12 units. Find the maximum possible area of a rectangle that can be placed inside the triangle with one side resting on the base of the triangle. What are the dimensions of the rectangle(s) of maximum area?

## Example 2



Example 2 A soft-drink manufacturer wants to fabricate cylindrical cans for its product. The can is to have a volume of 12 fluid ounces, which is approximately 22 cubic inches. Find the dimensions of the can that will require the least amount of material. See Figure 4.5.3.

## Example 3



Example 3 A window in the shape of a rectangle capped by a semicircle is to have perimeter $p$. Choose the radius of the semicircular part so that the window admits the greatest amount of light.

## Example 4



Example 4 A state highway department plans to construct a new road between towns $A$ and $B$. Town $A$ lies on an abandoned road that runs east-west. Town $B$ is 3 miles north of the point on that road that is 5 miles east of A . The engineering division proposes that the road be constructed by restoring a section of the old road from $A$ up to a point $P$ and joining it to a new road that connects $P$ and $B$. If the cost of restoring the old road is $\$ 200,000$ per mile and the cost of the new road is $\$ 400,000$ per mile, how much of the old road should be restored in order to minimize the department's costs?

## Example 5




Example 5 (The angle of incidence equals the angle of reflection.) Figure 4.5.6 depicts light from a point $A$ reflected to a point $B$ by a mirror. Two angles have been marked: the angle of incidence, $\theta_{i}$, and the angle of reflection, $\theta_{r}$. Experiment shows that $\theta_{i}=\theta_{r}$. Derive this result by postulating that the light that travels from $A$ to the mirror and then to $B$ follows the shortest possible path. $\dagger$

## Example 6

Example 6 A manufacturing plant has a capacity of 25 articles per week. Experience has shown that $n$ articles per week can be sold at a price of $p$ dollars each where $p=110-2 n$ and the cost of producing $n$ articles is $600+10 n+n^{2}$ dollars. How many articles should be made each week to give the largest profit?

| $n$ | $P$ | $n$ | $P$ | $n$ | $P$ |
| ---: | ---: | :---: | :---: | :---: | :---: |
| 8 | 8 | 14 | 212 | 20 | 200 |
| 9 | 57 | 15 | 225 | 21 | 177 |
| 10 | 100 | 16 | 232 | 22 | 148 |
| 11 | 137 | 17 | 233 | 23 | 113 |
| 12 | 168 | 18 | 228 | 24 | 72 |
| 13 | 193 | 19 | 217 | 25 | 25 |

