Lecture 14

Section 4.6 Concavity and Points of Inflection

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Test 1

- The written questions on Test 1 are graded and appear as a separate column in your CourseWare gradebook.
- You have to add the two columns "Test 1" and "FR1" to get your total score on the exam.
- The average in this class was 65.5!!! (Others 77.63, 75.01, 70.95)

Grade Information

- 90% and above - A
- at least 80% and below 90%- B
- at least 70% and below 80% - C
- at least 60% and below 70% - D
- below 60% - F

Grade Information

- 300 points determined by exams 1, 2 and 3
- 100 points determined by lab work, written quizzes, homework, daily grades and online quizzes.
- 200 points determined by the final exam
- 600 points total

Weekly Online Quizzes

- Online quizzes are given most weeks.
- You can attempt these quizzes as many times as you like until they expire.
- The highest grade will be used for your score.
- If you fail to reach 70% during three weeks of the semester, I have the option to drop you from the course!!!.

Quiz 1

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 2 or state that the value is not a critical value.

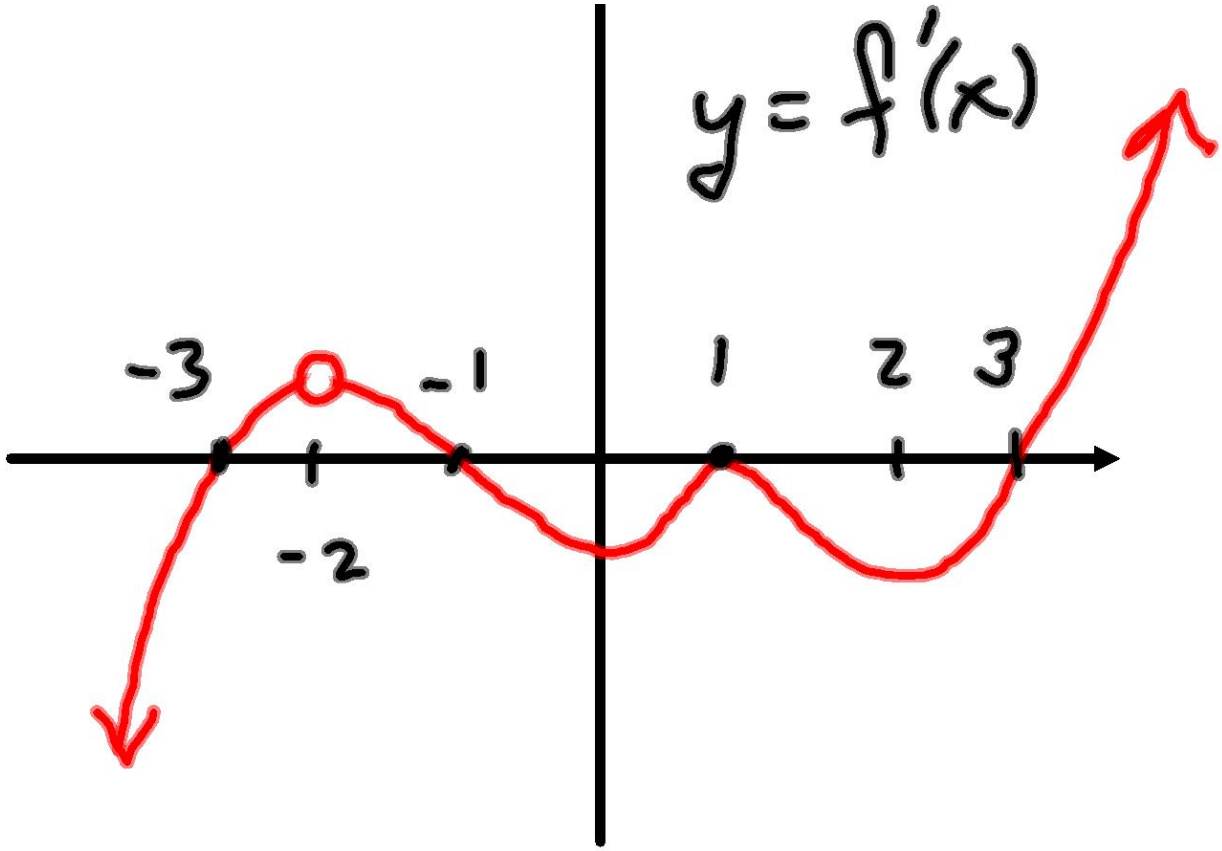
a. local maximum

b. local minimum

c. neither

d. not a critical value

e. None of these



Quiz 2

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 3 or state that the value is not a critical value.

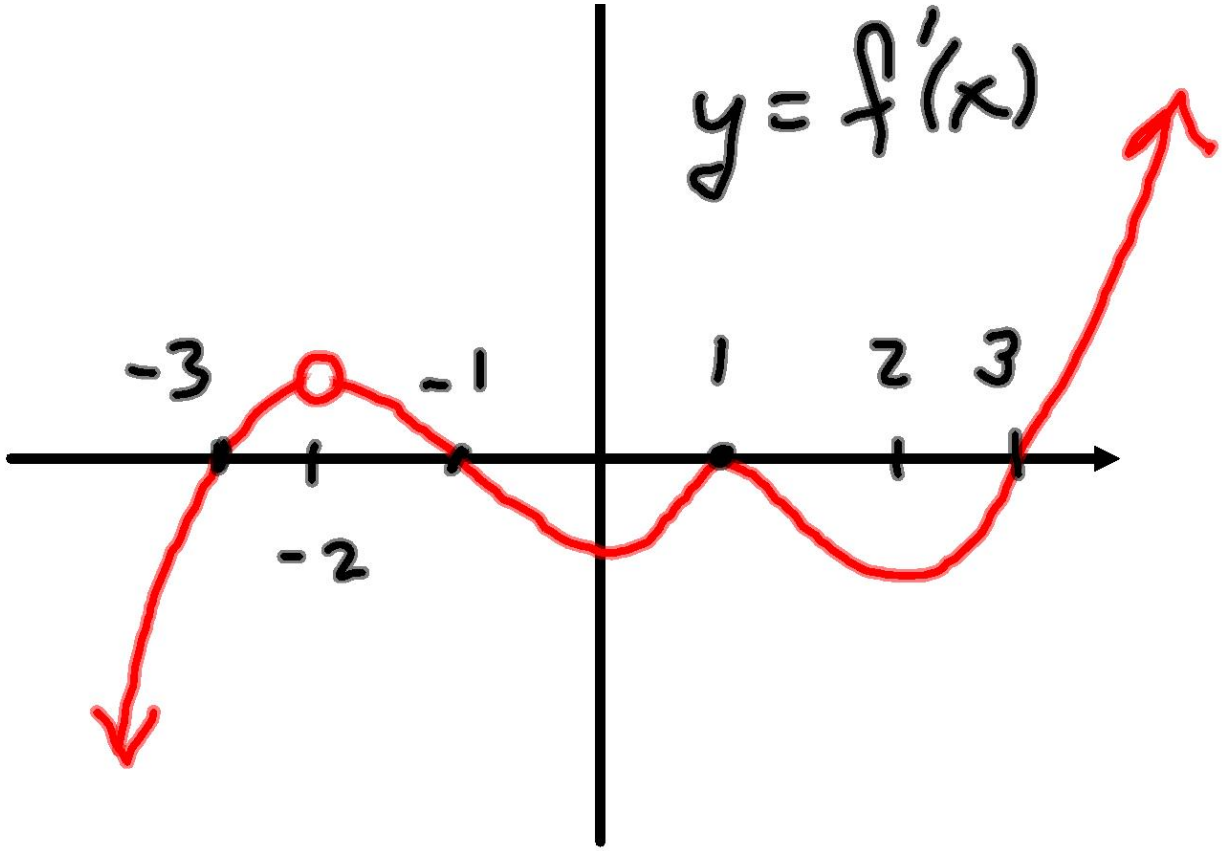
a. local maximum

b. local minimum

c. neither

d. not a critical value

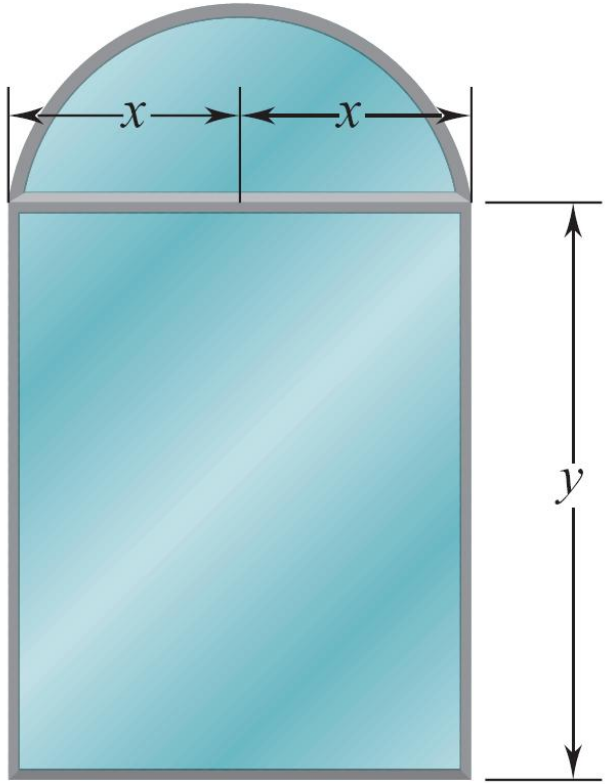
e. None of these



Homework Help Session

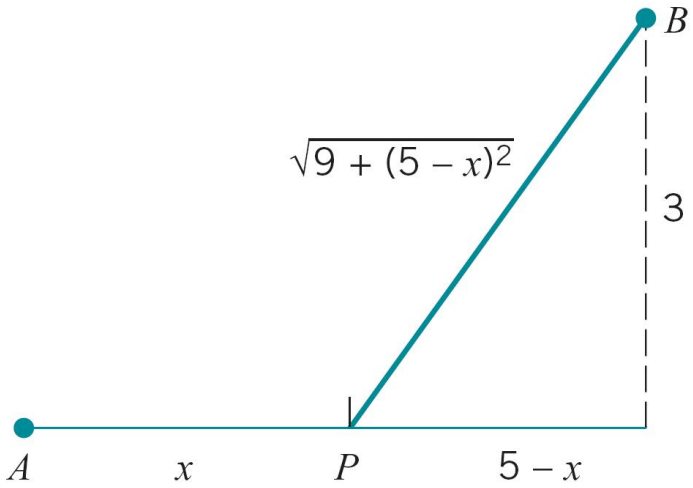
- Homework Help Session by by Prof. Morgan.
- Tonight 8:00 - 10:00pm in 100 SEC

Example 3



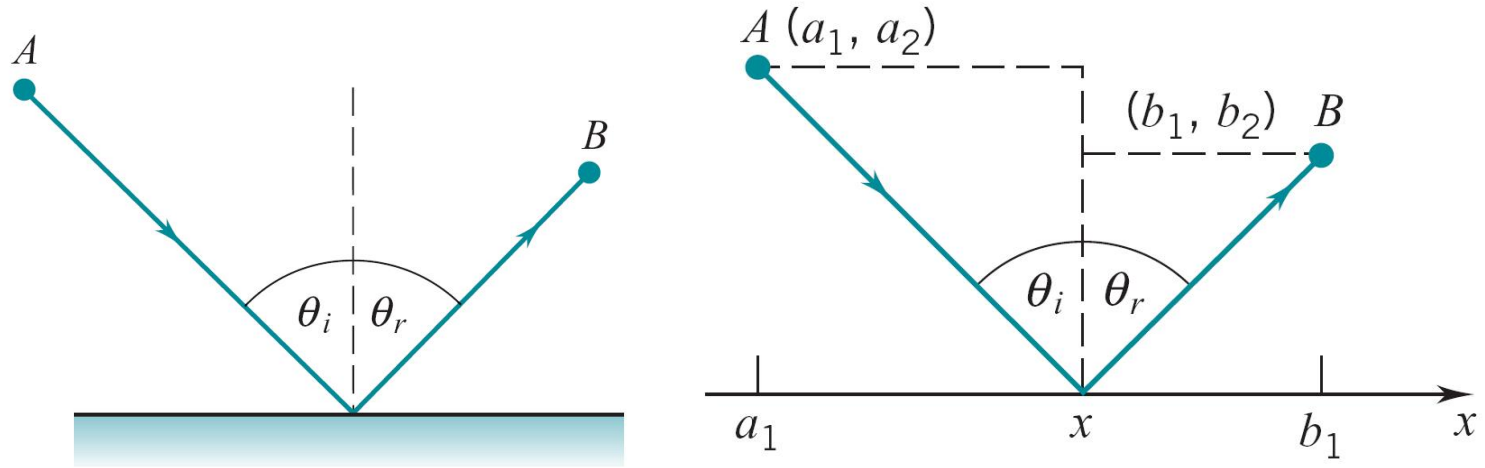
Example 3 A window in the shape of a rectangle capped by a semicircle is to have perimeter p. Choose the radius of the semicircular part so that the window admits the greatest amount of light.

Example 4



Example 4 A state highway department plans to construct a new road between towns A and B. Town A lies on an abandoned road that runs east-west. Town B is 3 miles north of the point on that road that is 5 miles east of A. The engineering division proposes that the road be constructed by restoring a section of the old road from A up to a point P and joining it to a new road that connects P and B. If the cost of restoring the old road is \$200,000 per mile and the cost of the new road is \$400,000 per mile, how much of the old road should be restored in order to minimize the department's costs?

Example 5



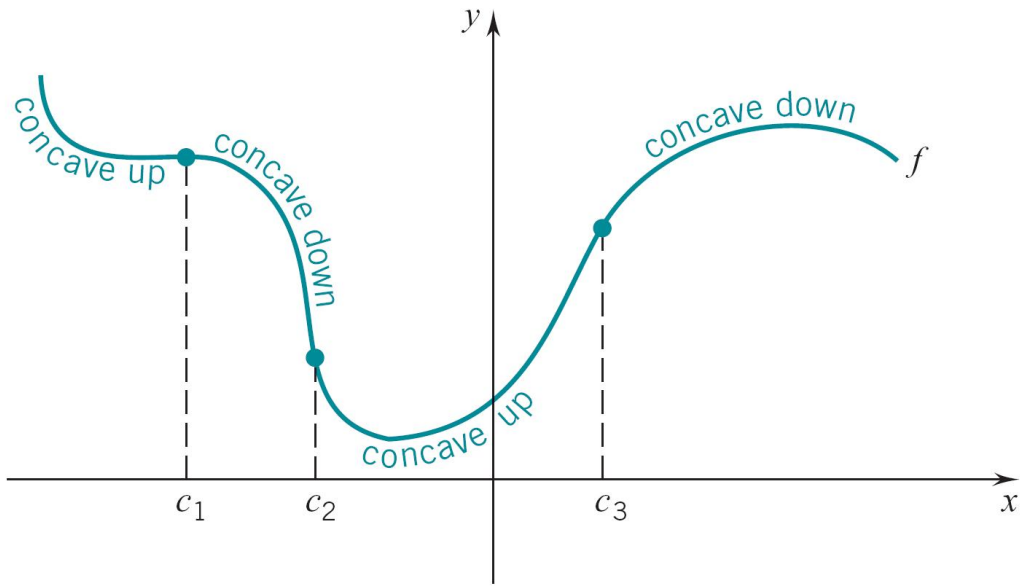
Example 5 (*The angle of incidence equals the angle of reflection.*) Figure 4.5.6 depicts light from a point A reflected to a point B by a mirror. Two angles have been marked: the *angle of incidence*, θ_i, and the *angle of reflection*, θ_r. Experiment shows that $\theta_i = \theta_r$. Derive this result by postulating that the light that travels from A to the mirror and then to B follows the shortest possible path.†

Example 6

Example 6 A manufacturing plant has a capacity of 25 articles per week. Experience has shown that n articles per week can be sold at a price of p dollars each where $p = 110 - 2n$ and the cost of producing n articles is $600 + 10n + n^2$ dollars. How many articles should be made each week to give the largest profit?

n	P	n	P	n	P
8	8	14	212	20	200
9	57	15	225	21	177
10	100	16	232	22	148
11	137	17	233	23	113
12	168	18	228	24	72
13	193	19	217	25	25

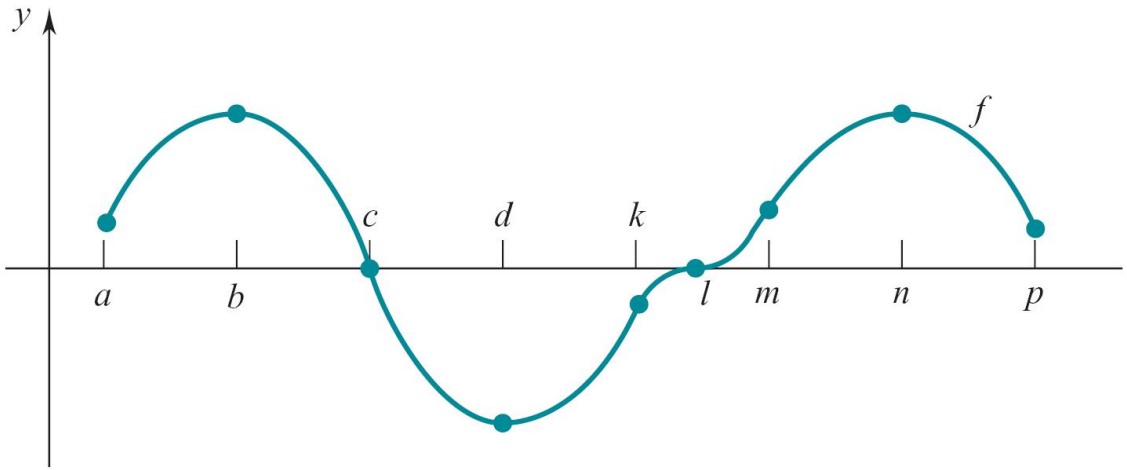
Concavity and Points of Inflection



Definition

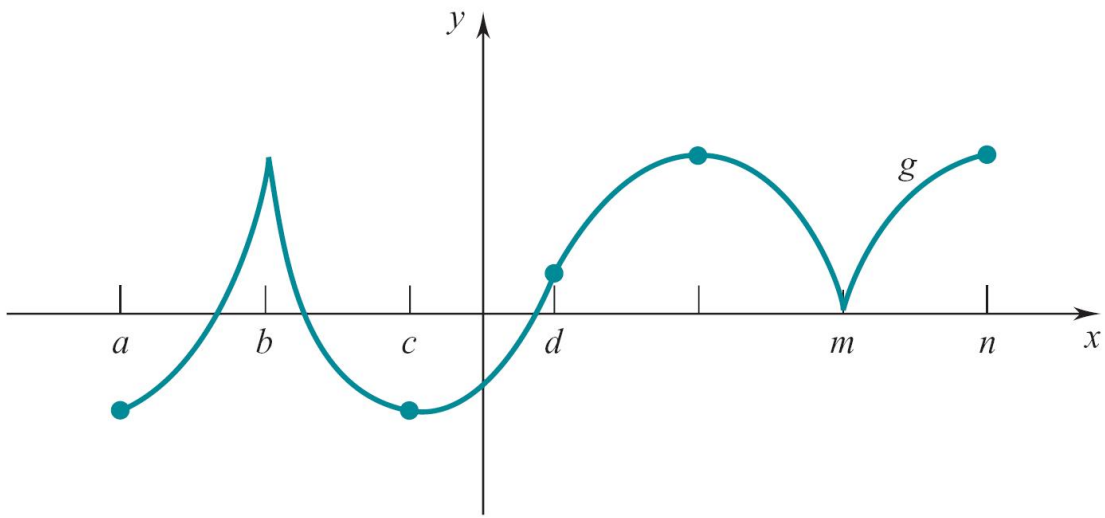
- The graph of f is concave up on I if f' increases on I.
- The graph of f is concave down on I if f' decreases on I.
- Ponts that join arcs of opposite concavity are points of inflection.

Example 1



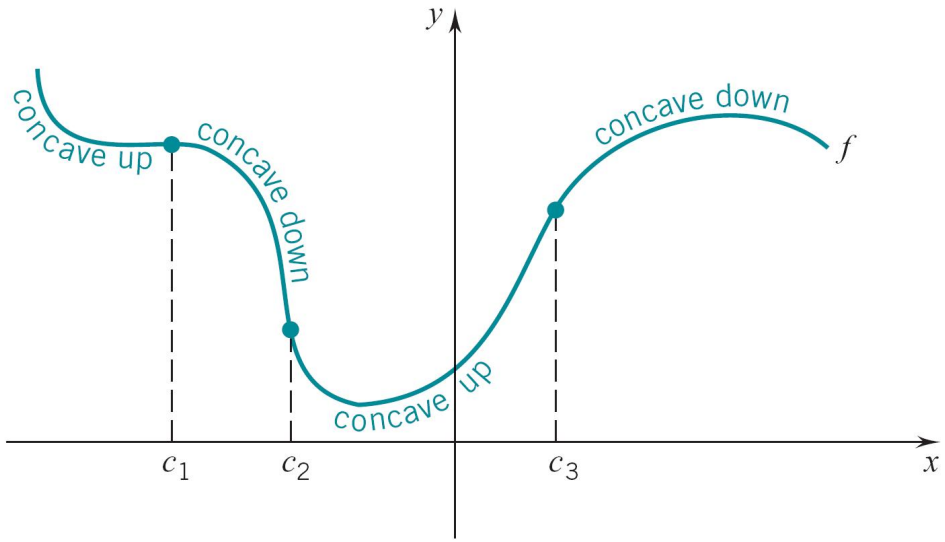
- Determine the intervals on which f increases and the intervals on which f decreases.
- Determine the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.
- Give the x-coordinates of the points of inflection.

Example 2



- Determine the intervals on which f increases and the intervals on which f decreases.
- Determine the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.
- Give the x-coordinates of the points of inflection.

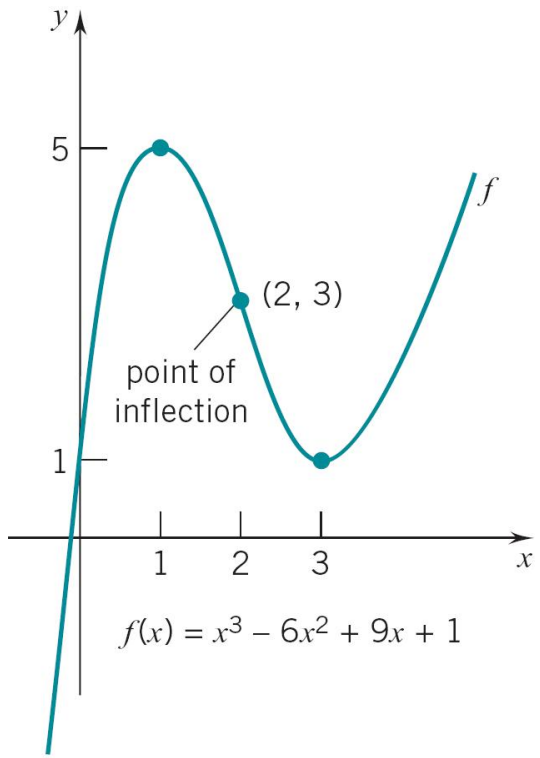
Second-Derivative Test



Theorem

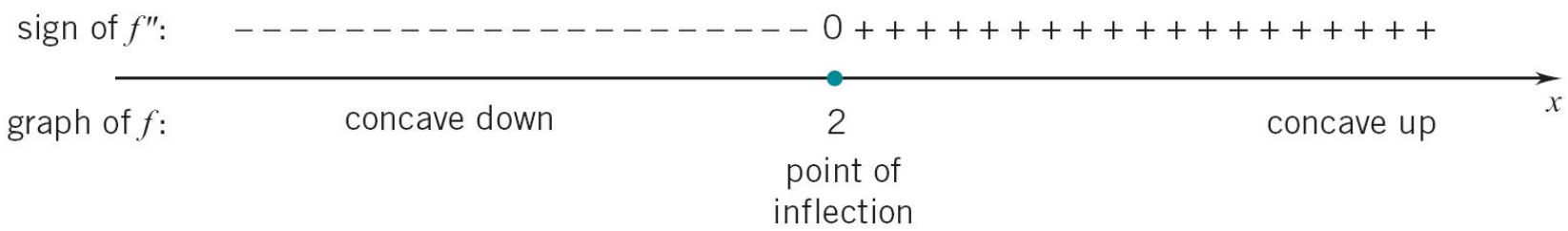
- *If $f''(x) > 0$ for all x in I, then f' increases on I, and the graph of f is concave up.*
- *If $f''(x) < 0$ for all x in I, then f' decreases on I, and the graph of f is concave down.*
- *If the point $(c, f(c))$ is a point of inflection, then either $f''(c) = 0$ or $f'(c)$ does not exist.*

Example 3

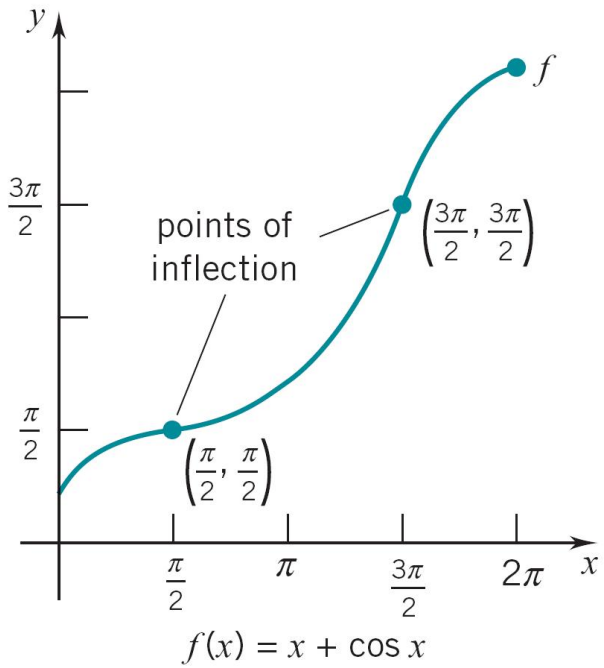


- Determine concavity and find the points of inflection of the graph of $f(x) = x^3 - 6x^2 + 9x + 1$.

$f'(x) = 3x^2 - 12x + 9$, $f''(x) = 6x - 12$.



Example 4



- Determine concavity and find the points of inflection of the graph of $f(x) = x + \cos x,\ x \in [0, 2\pi]$.

$f'(x) = 1 - \sin x,\ f''(x) = -\cos x.$