Lecture 14 Section 4.6 Concavity and Points of Inflection

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Test 1

- The written questions on Test 1 are graded and appear as a separate column in your CourseWare gradebook.
- You have to add the two columns "Test 1" and "FR1" to get your total score on the exam.
- The average in this class was 65.5!!! (Others 77.63, 75.01, 70.95)

Grade Information

- 90% and above - A
- at least 80% and below 90%- B
- at least 70% and below 80% - C
- at least 60% and below 70% - D
- below 60% - F

Grade Information

- 300 points determined by exams 1, 2 and 3
- 100 points determined by lab work, written quizzes, homework, daily grades and online quizzes.
- *200 points determined by the final exam*
- 600 points total

Weekly Online Quizzes

- Online quizzes are given most weeks.
- *You can attempt these quizzes as many times as you like until they expire.*
- The highest grade will be used for your score.
- If you fail to reach 70% during three weeks of the semester, I have the *option to drop you from the course!!!.*

Quiz 1

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 2 or state that the value is not a critical value.

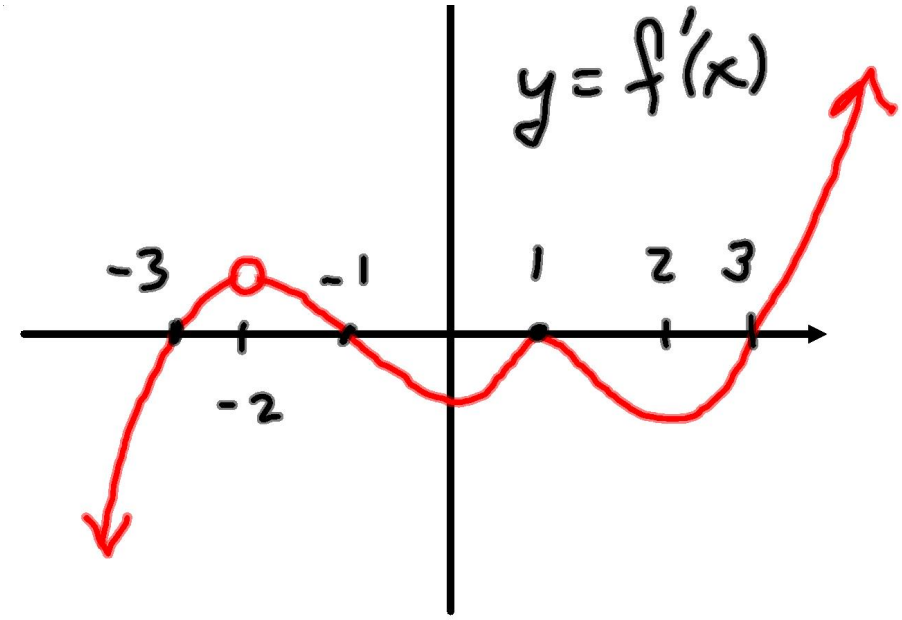
a. local maximum

b. local minimum

c. neither

d. not a critical value

e. None of these



Quiz 2

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 3 or state that the value is not a critical

value.

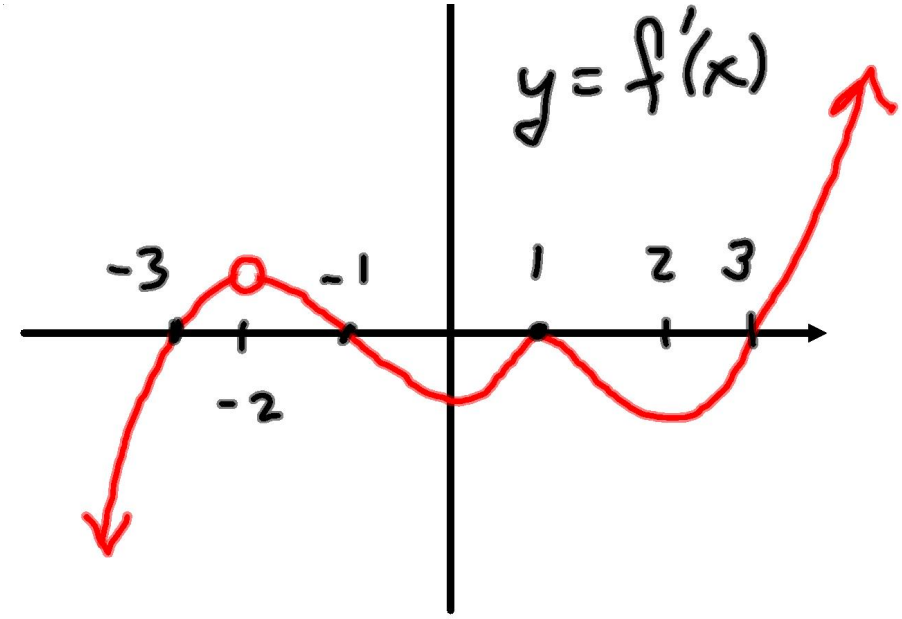
a. local maximum

b. local minimum

c. neither

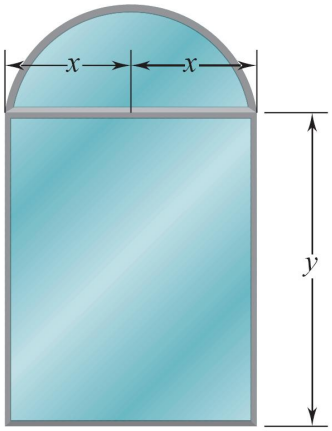
d. not a critical value

e. None of these



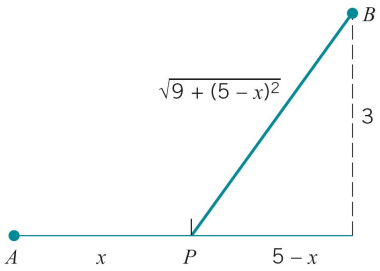
1 Section 4.5 Some Max-Min Problems (Cont.)

Example 3



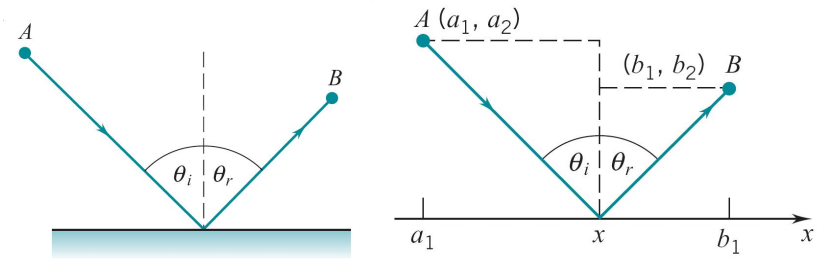
Example 3 A window in the shape of a rectangle capped by a semicircle is to have perimeter p. Choose the radius of the semicircular part so that the window admits the greatest amount of light.

Example 4



Example 4 A state highway department plans to construct a new road between towns A and B. Town A lies on an abandoned road that runs east-west. Town B is 3 miles north of the point on that road that is 5 miles east of A. The engineering division proposes that the road be constructed by restoring a section of the old road from A up to a point P and joining it to a new road that connects P and B. If the cost of restoring the old road is \$200,000 per mile and the cost of the new road is \$400,000 per mile, how much of the old road should be restored in order to minimize the department's costs?

Example 5



Example 5 (*The angle of incidence equals the angle of reflection.*) Figure 4.5.6 depicts light from a point A reflected to a point B by a mirror. Two angles have been marked: the *angle of incidence*, θ_i, and the *angle of reflection*, θ_r. Experiment shows that $\theta_i = \theta_r$. Derive this result by postulating that the light that travels from A to the mirror and then to B follows the shortest possible path.†

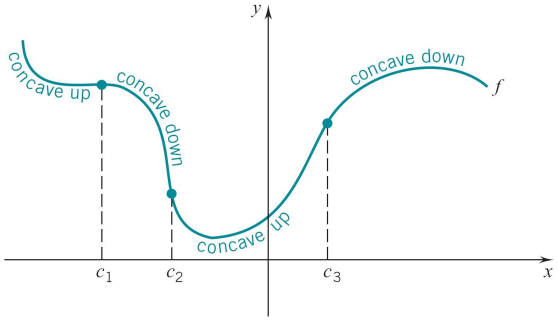
Example 6

Example 6 A manufacturing plant has a capacity of 25 articles per week. Experience has shown that n articles per week can be sold at a price of p dollars each where $p = 110 - 2n$ and the cost of producing n articles is $600 + 10n + n^2$ dollars. How many articles should be made each week to give the largest profit?

n	P	n	P	n	P
8	8	14	212	20	200
9	57	15	225	21	177
10	100	16	232	22	148
11	137	17	233	23	113
12	168	18	228	24	72
13	193	19	217	25	25

2 Section 4.6 Concavity and Points of Inflection

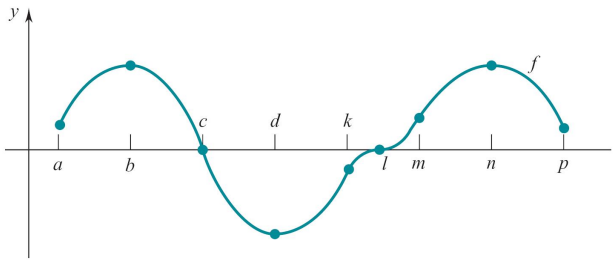
Concavity and Points of Inflection



Definition 1. • The graph of f is *concave up* on I if f' *increases* on I.

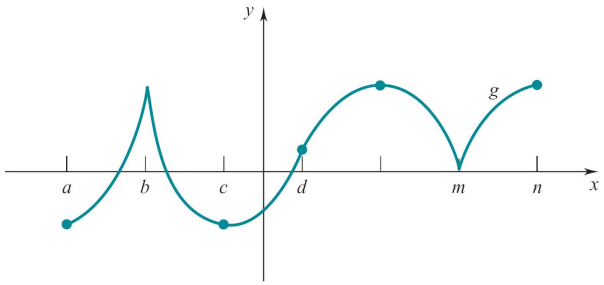
- The graph of f is *concave down* on I if f' *decreases* on I.
- Ponts that join arcs of *opposite concavity* are *points of inflection.*

Example 1



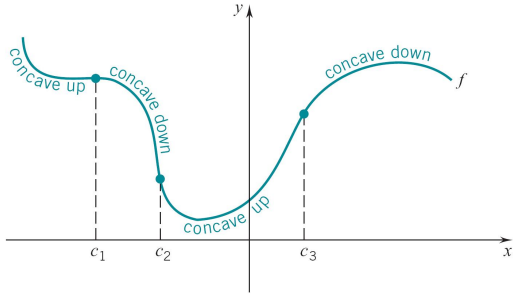
- Determine the intervals on which f increases and the intervals on which f decreases.
- Determine the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.
- Give the x-coordinates of the points of inflection.

Example 2



- Determine the intervals on which f increases and the intervals on which f decreases.
- Determine the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.
- Give the x-coordinates of the points of inflection.

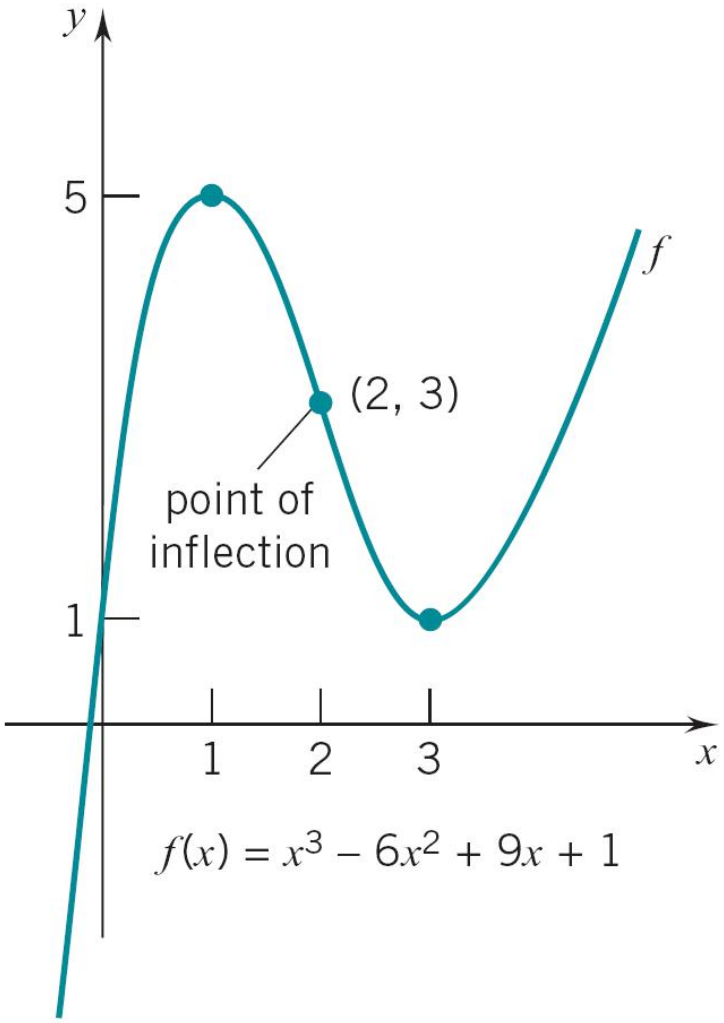
Second-Derivative Test



Theorem 2.

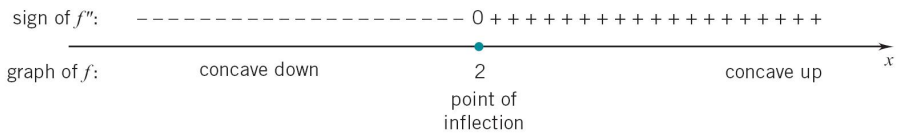
- *If $f''(x) > 0$ for all x in I, then f' increases on I, and the graph of f is concave up.*
- *If $f''(x) < 0$ for all x in I, then f' decreases on I, and the graph of f is concave down.*
- *If the point $(c, f(c))$ is a point of inflection, then either $f''(c) = 0$ or $f'(c)$ does not exist.*

Example 3

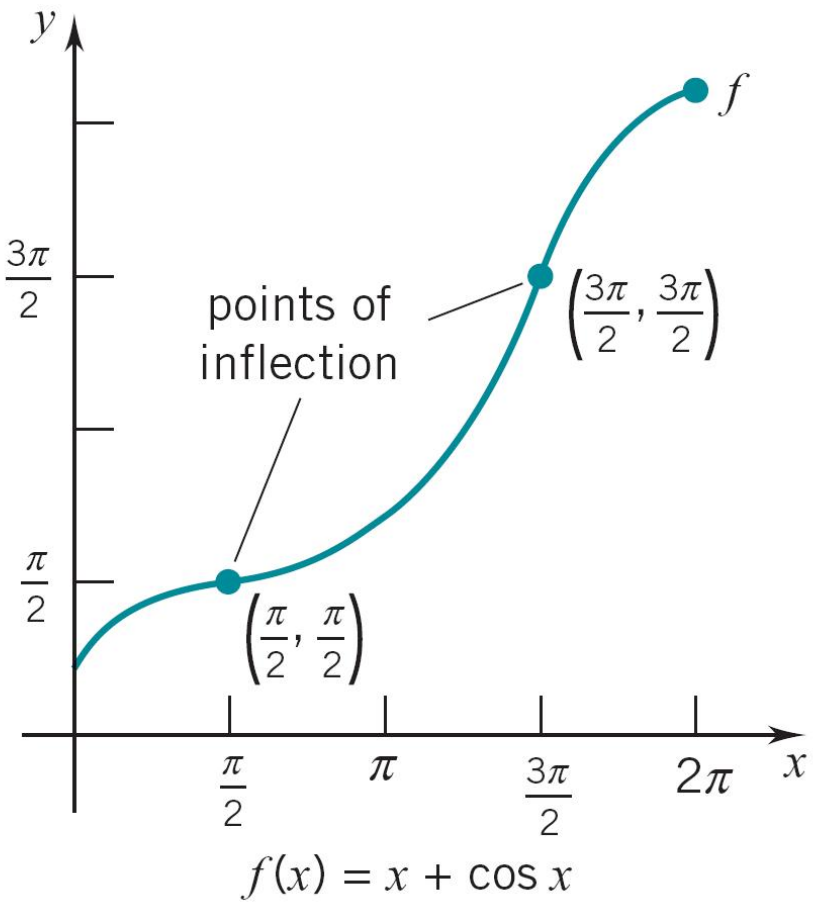


- Determine concavity and find the points of inflection of the graph of $f(x) = x^3 - 6x^2 + 9x + 1$.

$f'(x) = 3x^2 - 12x + 9,\ f''(x) = 6x - 12.$



Example 4



- Determine concavity and find the points of inflection of the graph of $f(x) = x + \cos x,\ x \in [0, 2\pi]$.

$f'(x) = 1 - \sin x,\ f''(x) = -\cos x.$