Lecture 15

Section 4.7 Vertical and Horizontal Asymptotes; Vertical Tangents and Cusps

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Test 2

- Test 2: November 1-4 in CASA
- Loggin to CourseWare to reserve your time to take the exam.

Review for Test 2

- Review for Test 2 by the College Success Program.
- Friday, October 24 2:30–3:30pm in the basement of the library by the C-site.

Grade Information

- 300 points determined by exams 1, 2 and 3
- 100 points determined by lab work, written quizzes, homework, daily grades and online quizzes.
- 200 points determined by the final exam
- 600 points total

More Grade Information

- 90% and above - A
- at least 80% and below 90%- B
- at least 70% and below 80% - C
- at least 60% and below 70% - D
- below 60% - F

Online Quizzes

- Online Quizzes are available on CourseWare.
- If you fail to reach 70% during three weeks of the semester, I have the option to drop you from the course!!!.

Dropping Course

- Tuesday, November 4, 2008
- Last day to drop a course or withdraw with a "W" (must be by 5 pm)

Quiz 1

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of critical values of f.

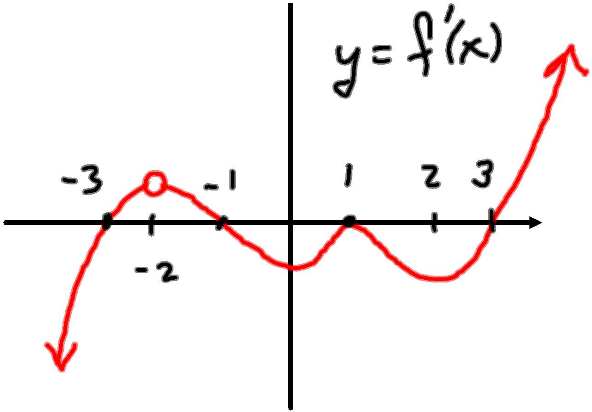
a. 2

b. 3

c. 4

d. 5

e. None of these



Quiz 2

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals of increase of f.

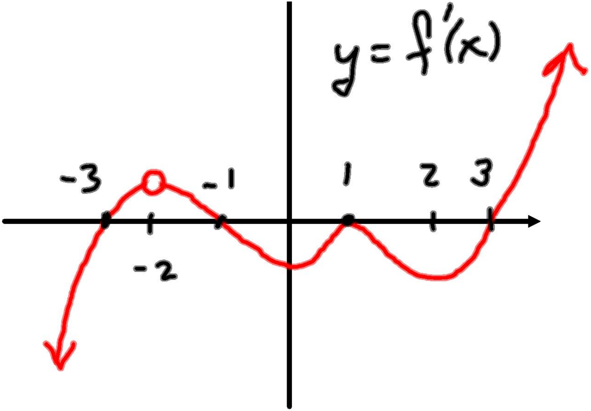
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 3

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals of decrease of f.

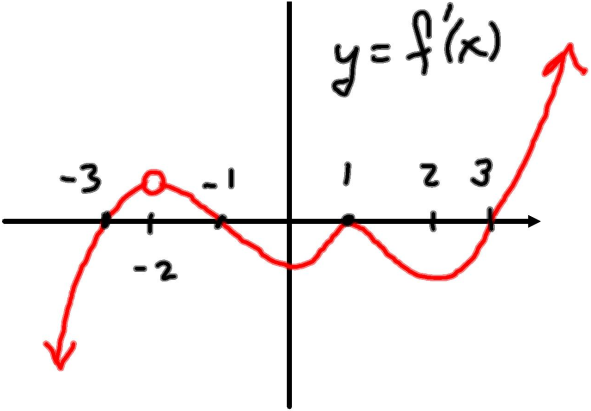
a. 1

b. 2

c. 3

d. 4

e. None of these



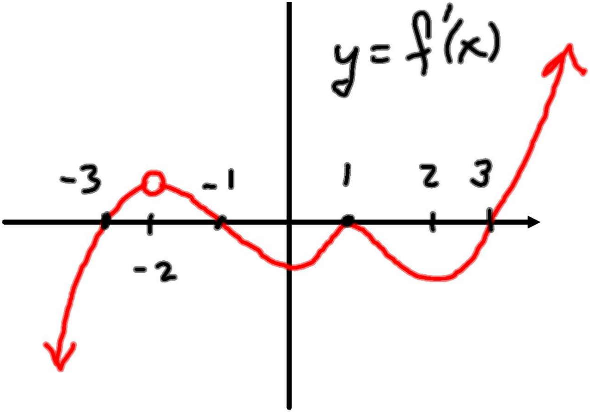
Quiz 4

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the smallest critical number of f

a. local maximum

b. local minimum

c. neither



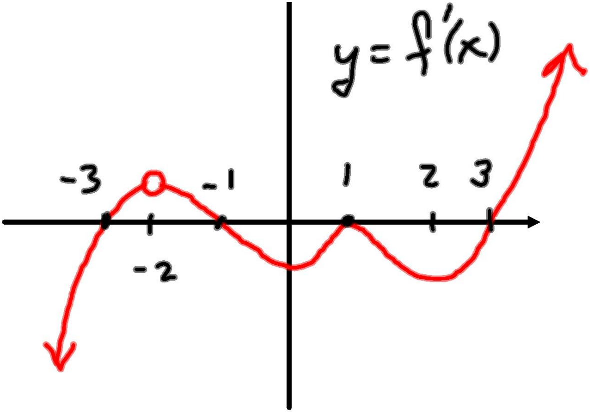
Quiz 5

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical number of f between 0 and 2.

a. local maximum

b. local minimum

c. neither



Quiz 6

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals where the graph of f is concave up.

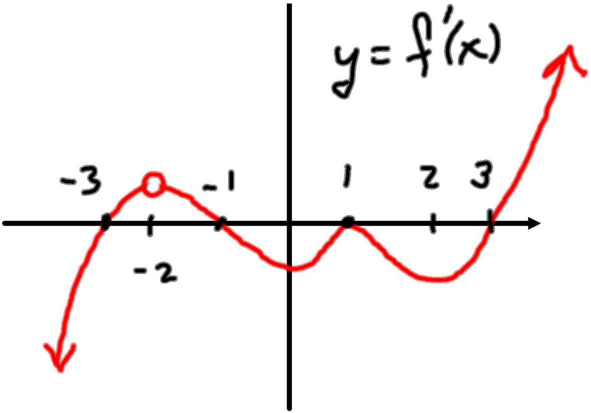
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 7

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals where the graph of f is concave down.

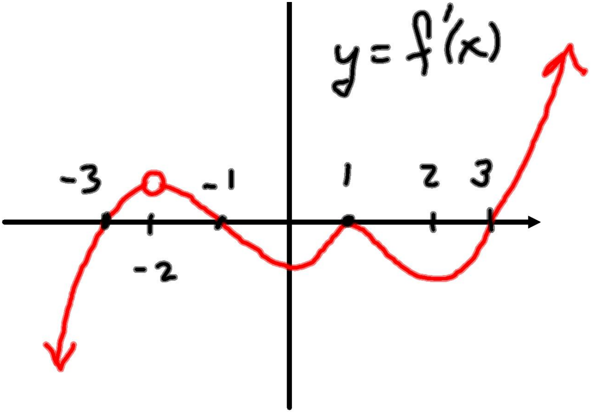
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 8

Assume the domain of f is all real numbers. The graph of $f'(x)$ is shown below. Give the number of the points of inflection of the graph of f.

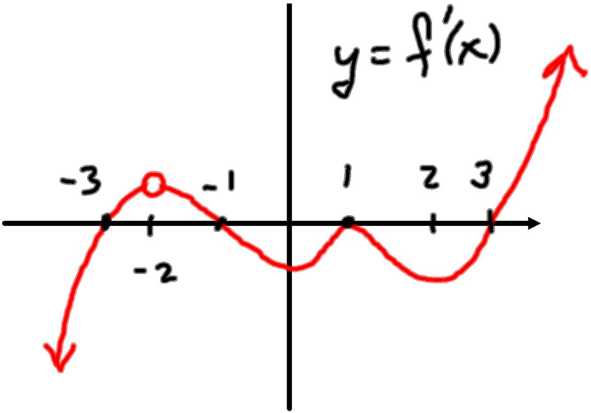
a. 1

b. 2

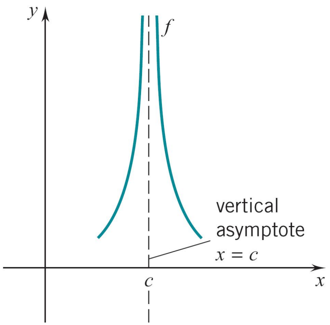
c. 3

d. 4

e. None of these



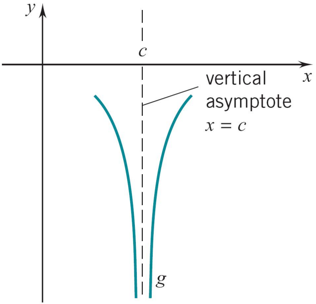
Vertical Aymptotes: Example 1



The line $x = c$ is a vertical asymptote for the function f:

$$f(x) \to \infty \quad \text{as } x \to c.$$

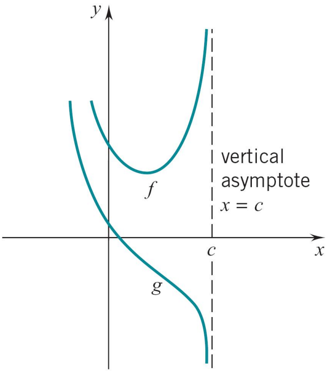
Vertical Aymptotes: Example 2



The line $x = c$ is a vertical asymptote for the function f:

$$f(x) \to -\infty \quad \text{as } x \to c.$$

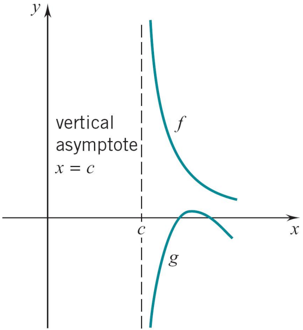
Vertical Aymptotes: Example 3



The line $x = c$ is a vertical asymptote for both functions f and g:

$$f(x) \to \infty \text{ and } g(x) \to -\infty \quad \text{as } x \to c^{-}.$$

Vertical Aymptotes: Example 4



The line $x = c$ is a vertical asymptote for both functions f and g:

$$f(x) \to \infty \text{ and } g(x) \to -\infty \qquad \text{as } x \to c^{+}.$$

How to locate Vertical Aymptotes

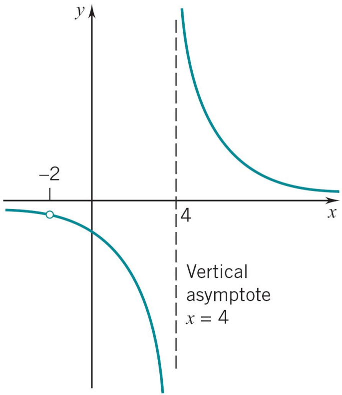
Typically, to locate the vertical asymptotes for a function f,

- find the values $x = c$ at which f is discontinuous
- and determine the behavior of f as x approaches c.

The vertical line $x = c$ is a vertical asymptote for f if any one of the following conditions holds

- $f(x) \to \infty$ or $-\infty$ as $x \to c^+$;
- $f(x) \to \infty$ or $-\infty$ as $x \to c^-$;
- $f(x) \to \infty$ or $-\infty$ as $x \to c$.

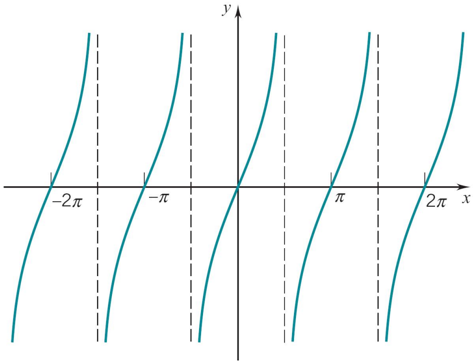
Vertical Aymptotes: Rational Function



The line $x = 4$ is a vertical asymptote for

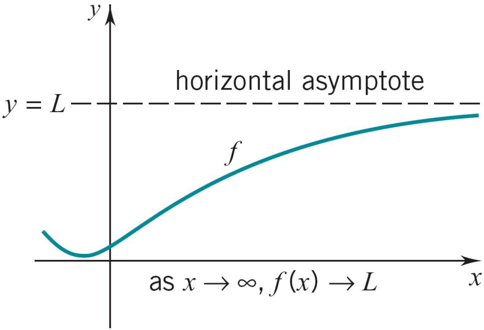
$$f(x) = \frac{3x + 6}{x^2 - 2x - 8} = \frac{3(x + 2)}{(x + 2)(x - 4)}.$$

Vertical Aymptotes: Tangent Function



The line $x = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \cdots$, are vertical asymptotes for the tangent function.

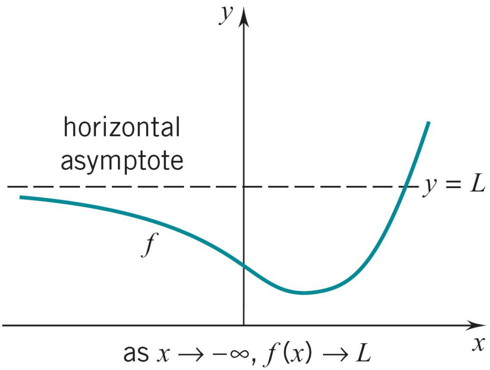
Horizontal Aymptote: Example 1



The line $y = L$ is a horizontal asymptote for the function f:

$$f(x) \to L \quad \text{as } x \to \infty.$$

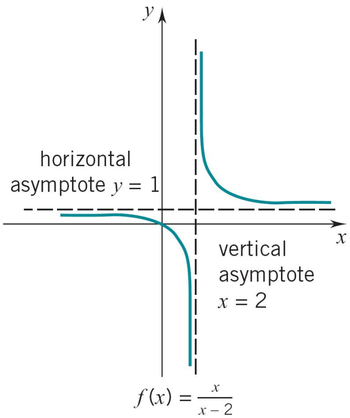
Horizontal Aymptote: Example 2



The line $y = L$ is a horizontal asymptote for the function f:

$$f(x) \to L \quad \text{as } x \to -\infty.$$

Aymptotes: Rational Function $f(x) = \frac{x}{x-2}$



- The line $x = 2$ is a vertical asymptote.
- The line $y = 1$ is a horizontal asymptote.

Behavior of Rational Function as $x \to \pm\infty$

Let

$$R(x) = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_k x^k + \cdots + b_1 x + b_0}$$

be a rational function. Then

- if $n < k$,

$$R(x) \to 0 \quad \text{as} \quad x \to \pm\infty;$$

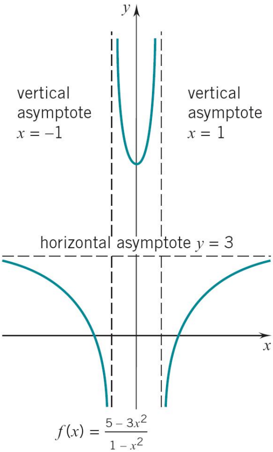
- if $n = k$,

$$R(x) \to \frac{a_n}{b_n} \quad \text{as} \quad x \to \pm\infty;$$

- if $n > k$,

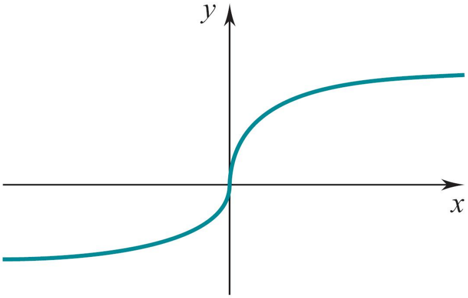
$$R(x) \to \pm\infty \quad \text{as} \quad x \to \pm\infty.$$

Aymptotes: Rational Function $f(x) = \frac{5-3x^2}{1-x^2}$



- The lines $x = \pm 1$ are vertical asymptotes.
- The line $y = 3$ is a horizontal asymptote.

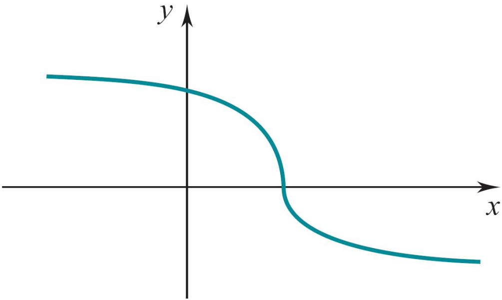
Vertical Tangent: Rational Power $f(x) = x^{1/3}$



The graph of $f(x) = x^{1/3}$ has a vertical tangent at the point $(0, 0)$ since

$$f'(x) = \frac{1}{3}x^{-2/3} \to \infty \quad \text{as } x \to 0.$$

Vertical Tangent: Rational Power $g(x) = (2 - x)^{1/5}$

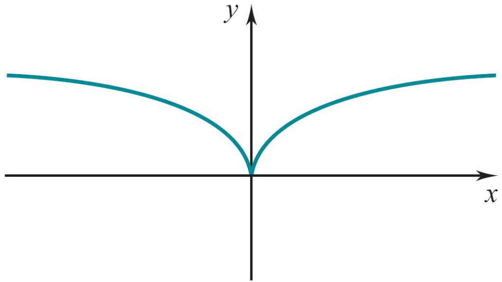


$g(x) = (2 - x)^{1/5}$

The graph of $g(x) = (2 - x)^{1/5}$ has a vertical tangent at the point $(2, 0)$ since

$$g'(x) = -\frac{1}{5}(2-)x^{-4/5} \rightarrow -\infty \quad \text{as } x \rightarrow 2.$$

Vertical Cusp: Rational Power $f(x) = x^{2/3}$

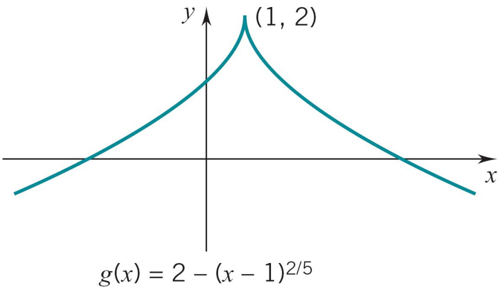


$f(x) = x^{2/3}$

The graph of $f(x) = x^{2/3}$ has a vertical cusp at the point $(0, 0)$ since $f'(x) = \frac{2}{3}x^{-1/3}$ and

$$f'(x) \to -\infty \text{ as } x \to 0^-, \quad \text{and} \quad f'(x) \to \infty \text{ as } x \to 0^+.$$

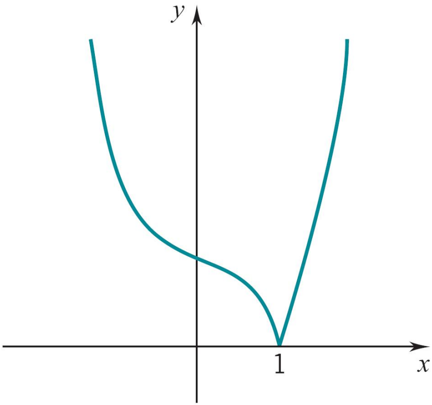
Vertical Cusp: Rational Power $g(x) = 2 - (x-1)^{2/5}$



The graph of $g(x) = 2 - (x-1)^{2/5}$ has a vertical cusp at the point $(1,2)$ since $g'(x) = -\frac{2}{5}(x-1)^{-3/5}$ and

$$g'(x) \to \infty \text{ as } x \to 0^-, \quad \text{and} \quad g'(x) \to -\infty \text{ as } x \to 0^+.$$

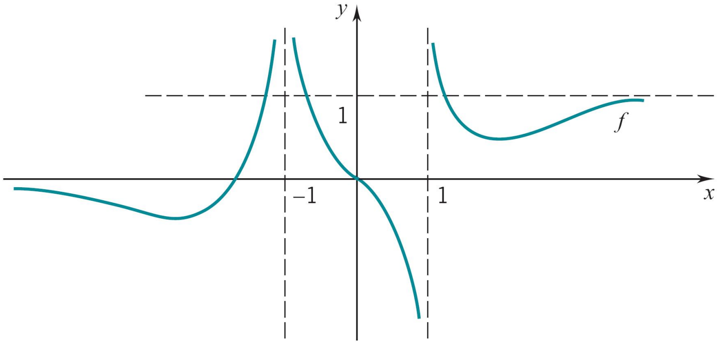
Example: $f(x) = |x^3 - 1|$



$f(x) = |x^3 - 1|$

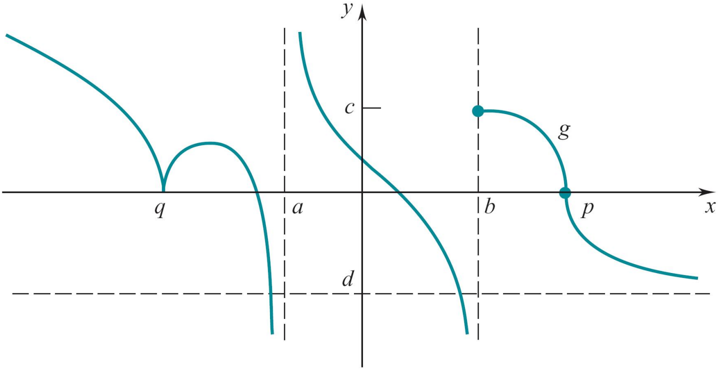
Is there a vertical cusp for the graph of $f(x) = |x^3 - 1|$?

Example



- Give the equations of the vertical asymptotes, if any.
- Give the equations of the horizontal asymptotes, if any.

Example



- Give the equations of the vertical asymptotes, if any.
- Give the equations of the horizontal asymptotes, if any.
- Give the number c, if any, at which the graph has a vertical cusp.